CE 191: Civil and Environmental Engineering Systems Analysis

LEC 03 : Graphical Solutions to LP

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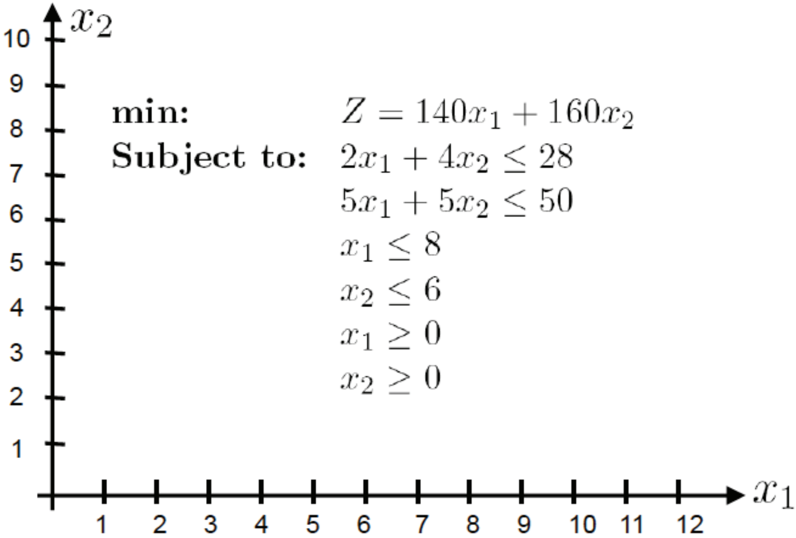
Civil & Environmental Engineering
University of California, Berkeley

Fall 2014

Example:

$$
\begin{aligned}
\min \quad & J = 140x_1 + 160x_2 \\
\text{s. to} \quad & 2x_1 + 4x_2 \leq 28 \\
& 5x_1 + 5x_2 \leq 50 \\
& x_1 \leq 8 \\
& x_2 \leq 6 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

Construction of the feasible set



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\begin{aligned}
\textbf{min:} \quad & Z = 140x_1 + 160x_2 \\
\textbf{Subject to:} \quad & 2x_1 + 4x_2 \leq 28 \\
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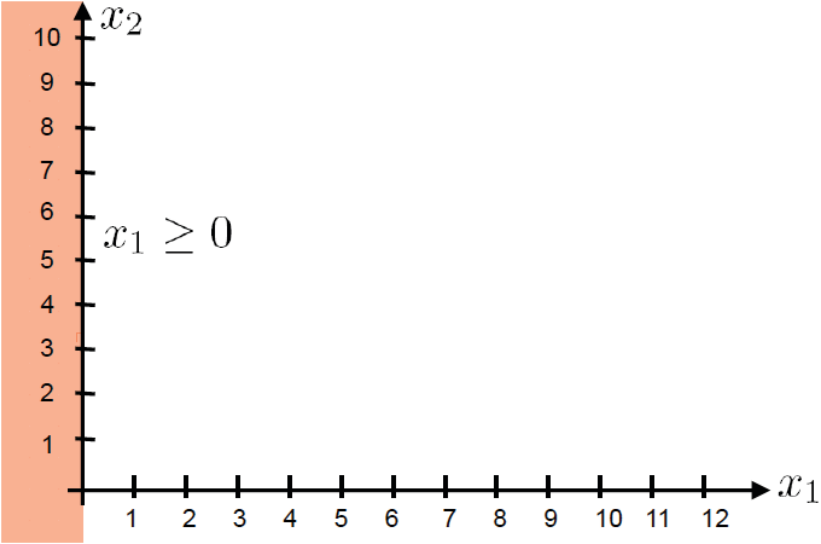

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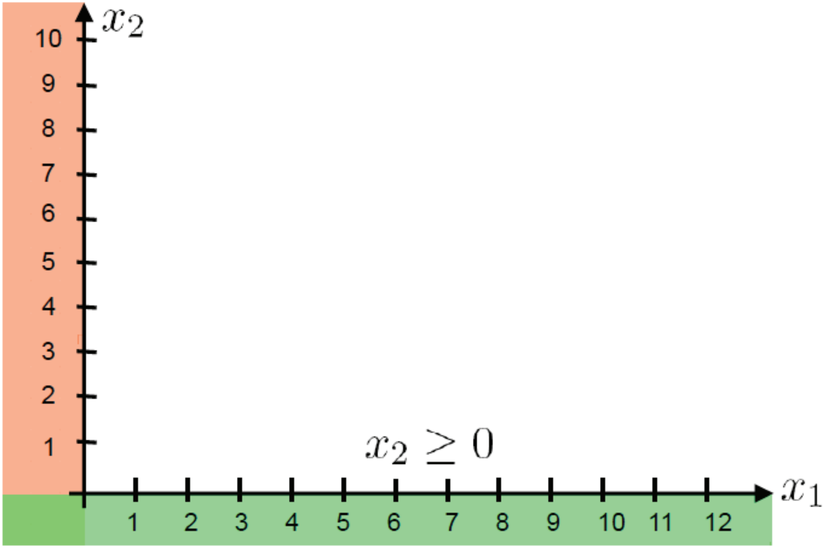

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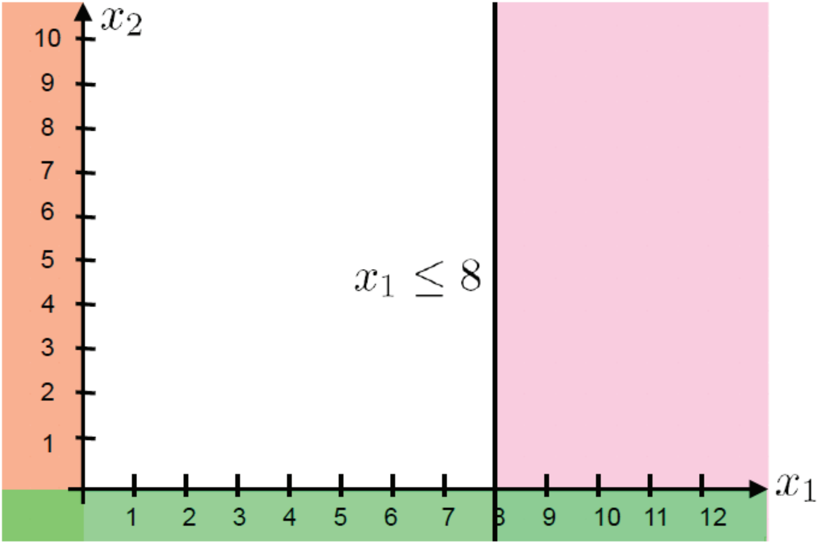

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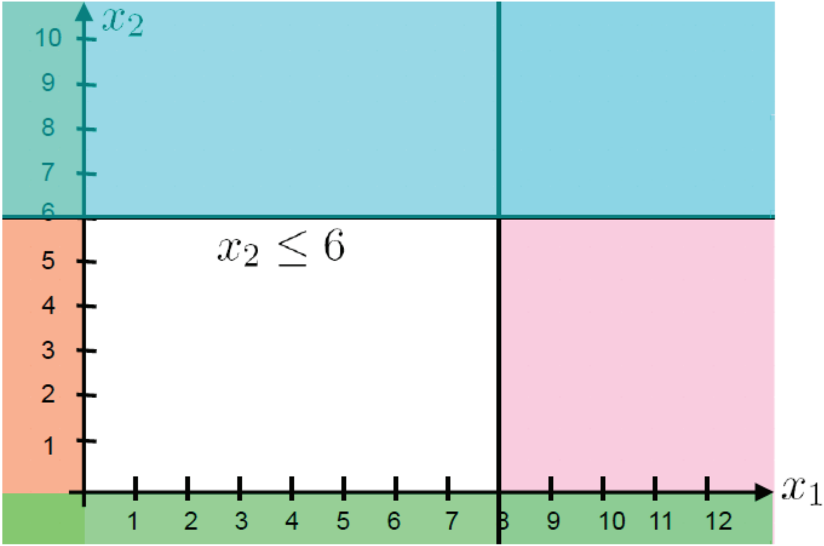

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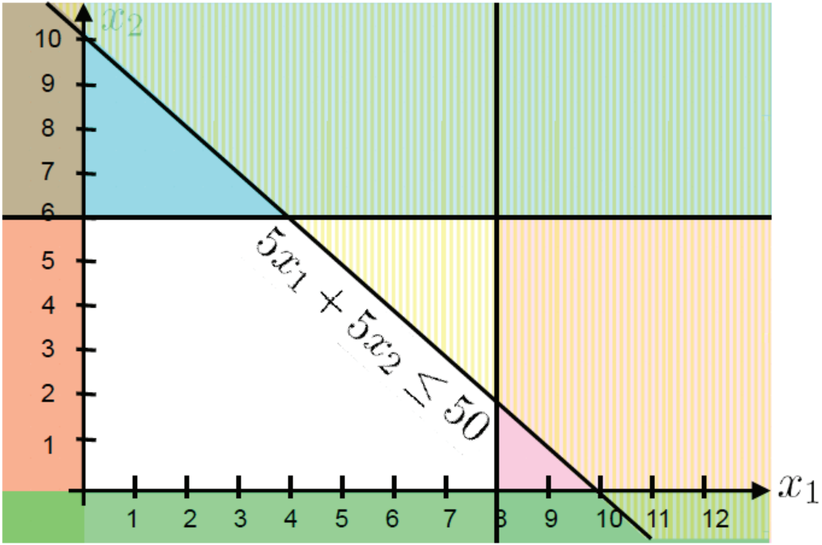

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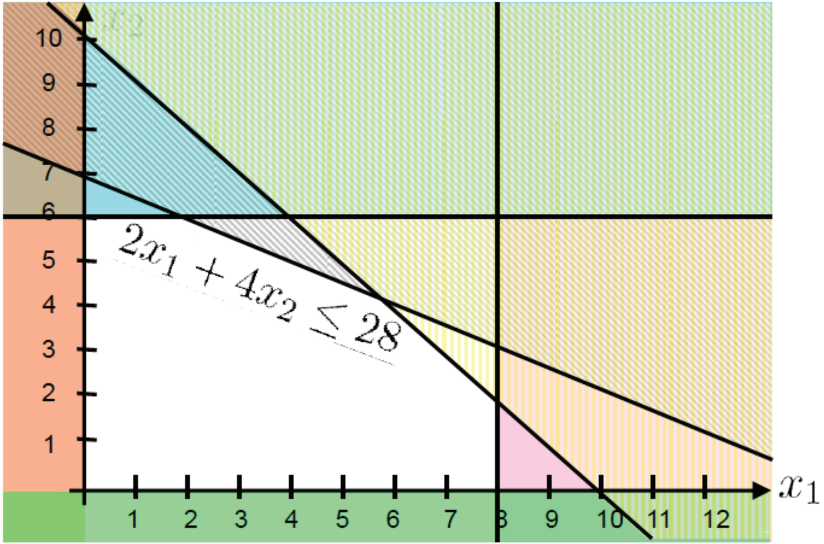
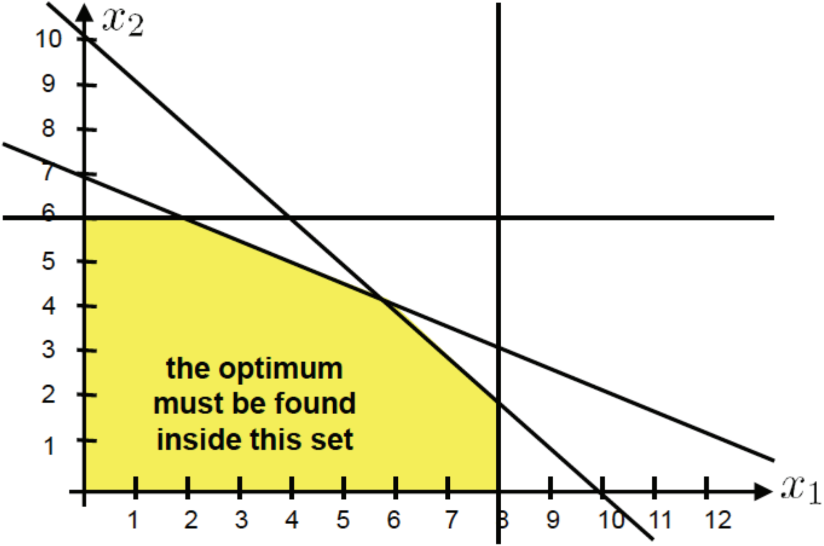
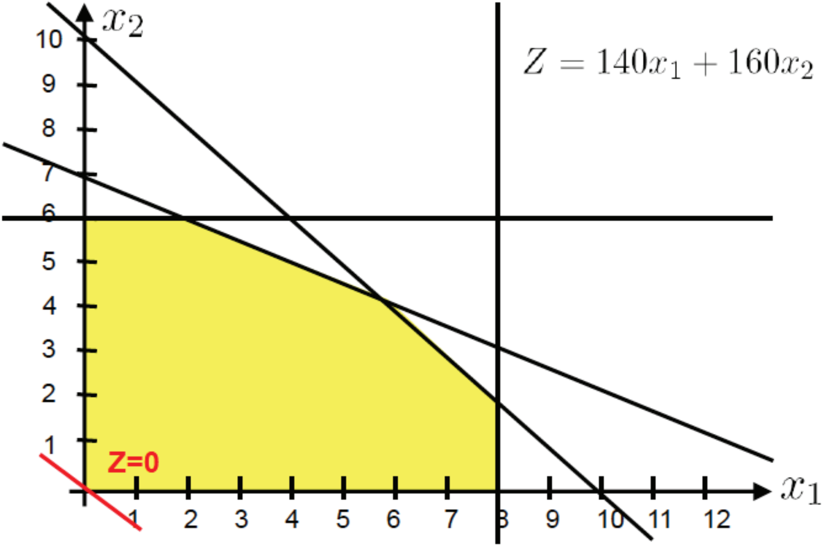


Feasible Set Final Result



Isolines



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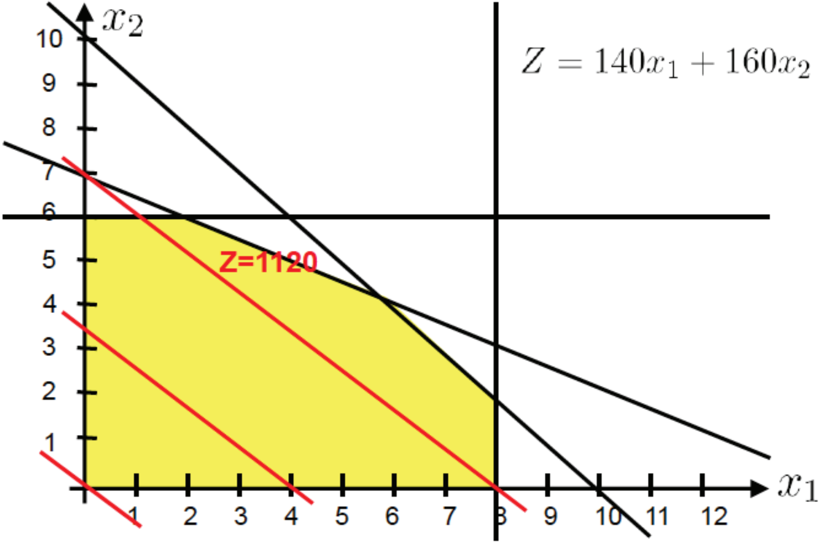
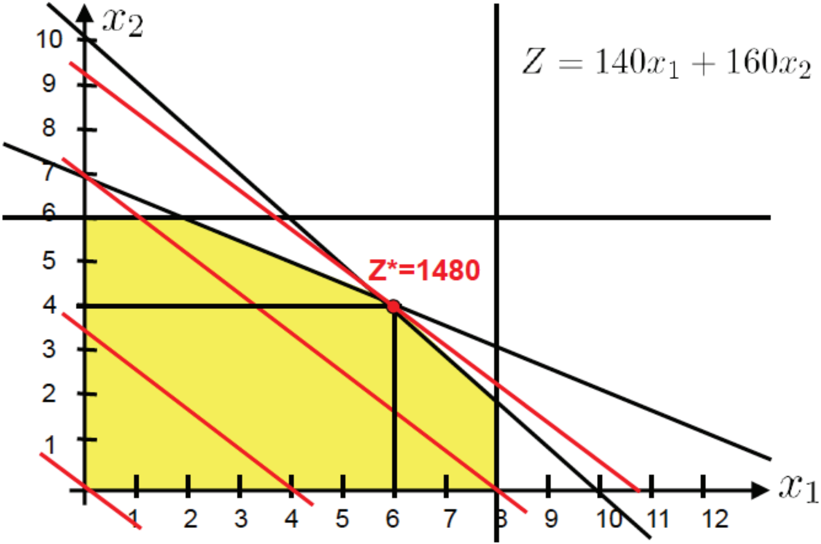
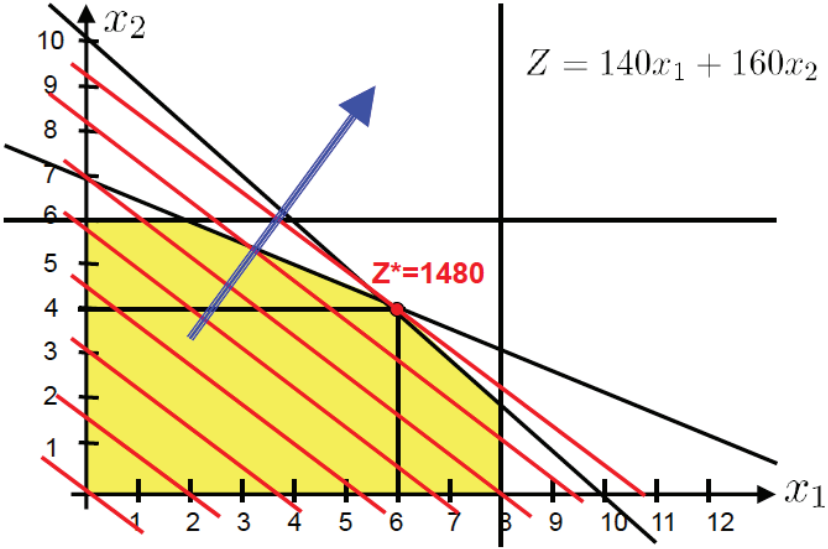


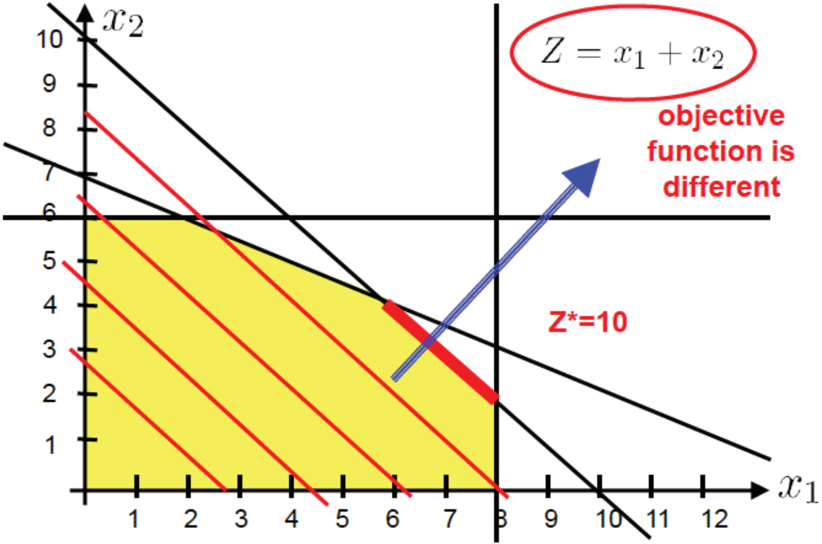
Isolines



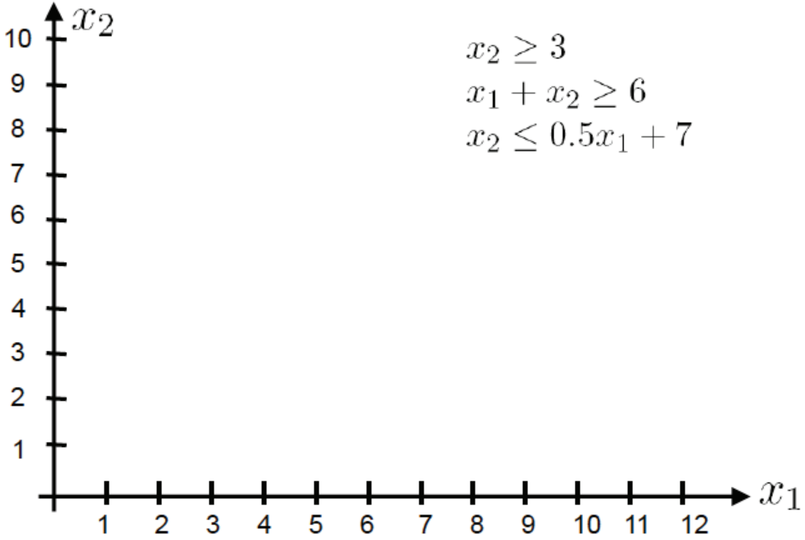
Gradient of the cost function



Uniqueness (or not) of the cost function

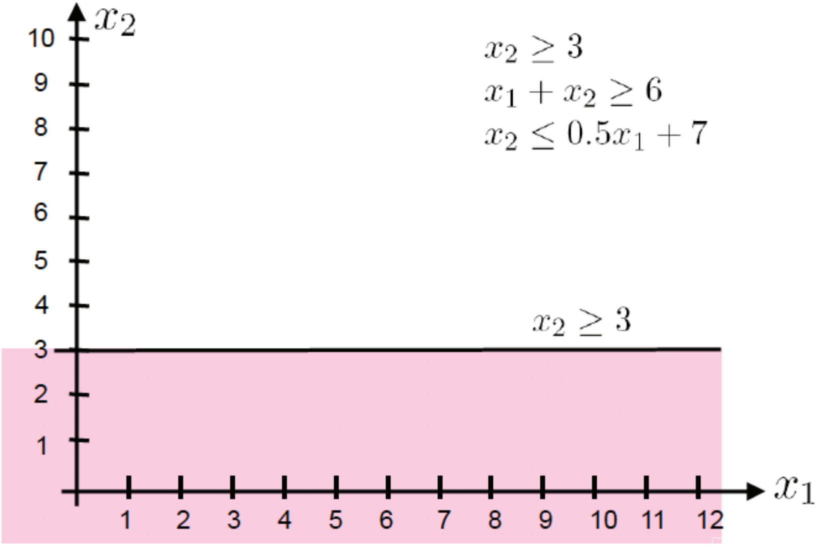


Features of the feasible set



$$x_2 \geq 3$$
$$x_1 + x_2 \geq 6$$
$$x_2 \leq 0.5x_1 + 7$$

Features of the feasible set



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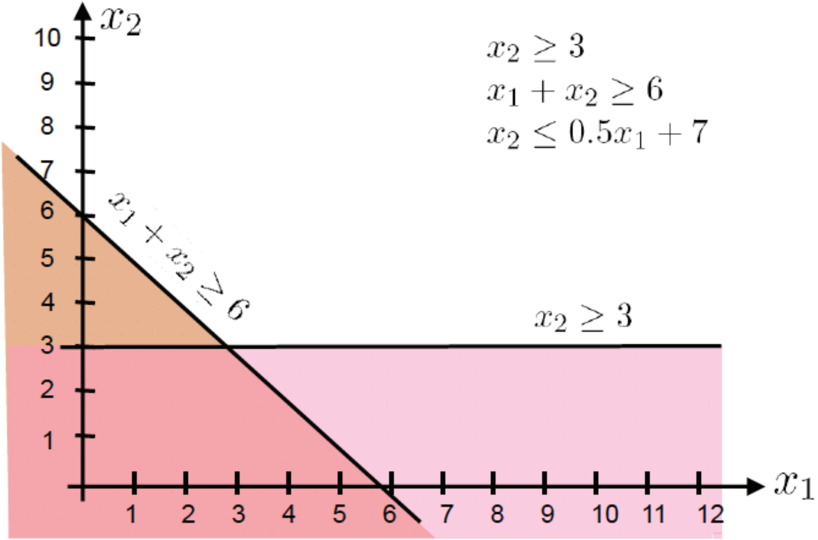

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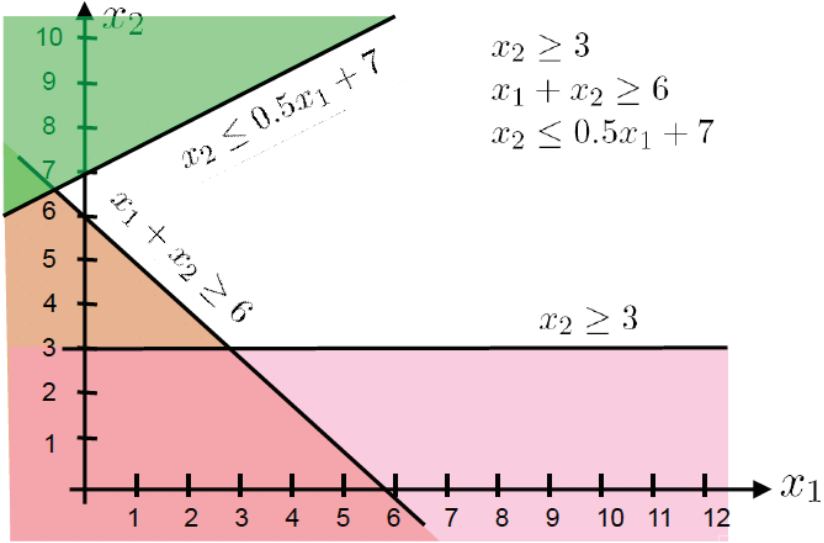

$$x_2 \geq 3$$
$$x_1 + x_2 \geq 6$$
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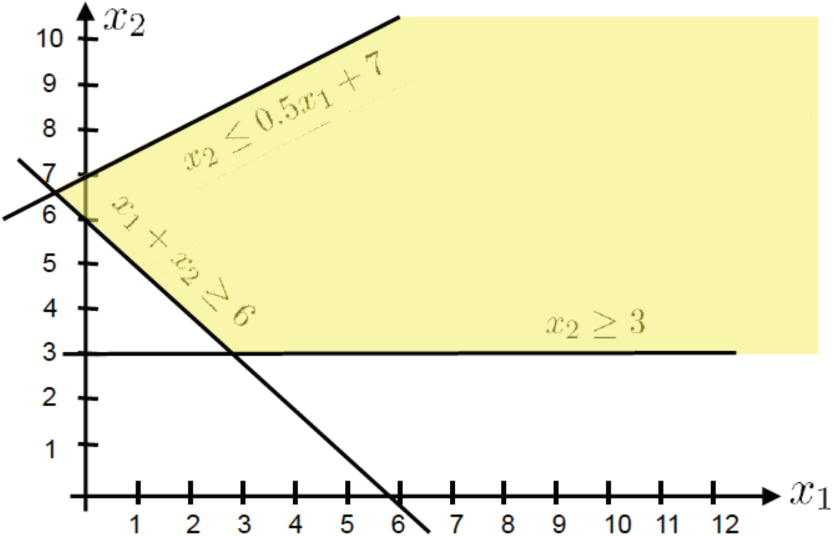
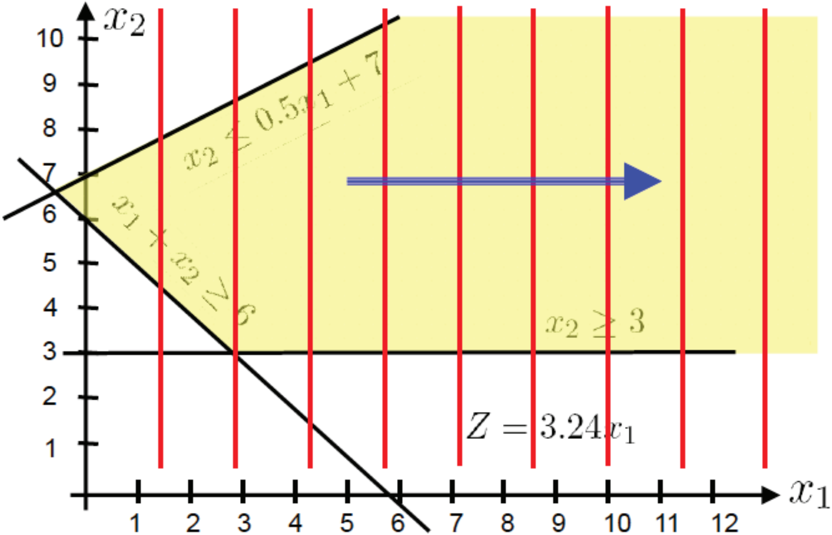
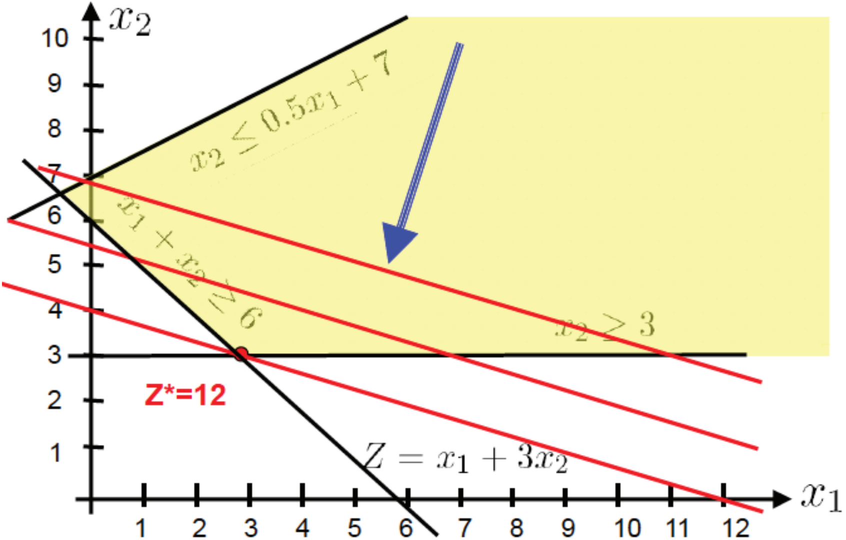
Feasible set is unbounded



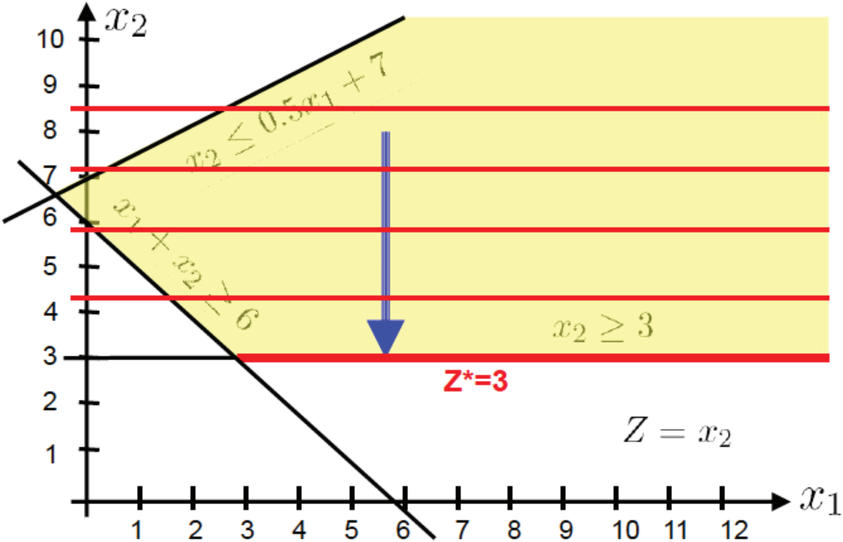
Objective function might be unbounded too



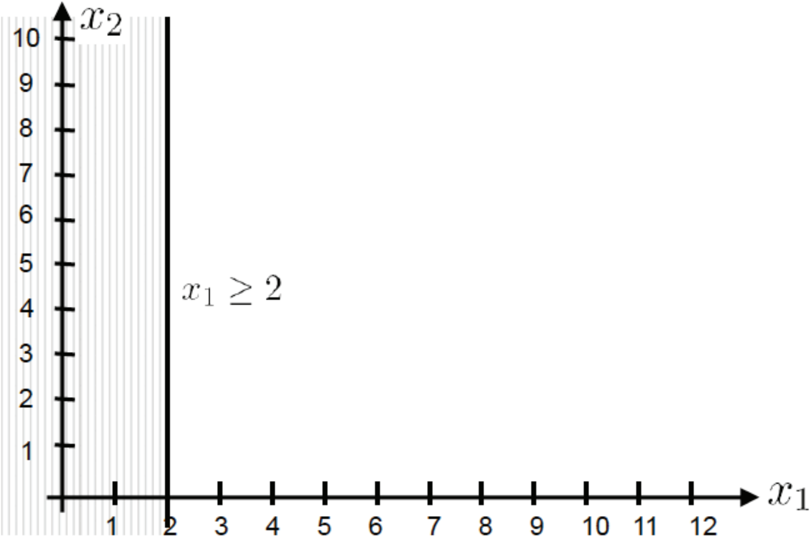
Objective function might be bounded



Optimum may be non-unique



Feasible set might be empty



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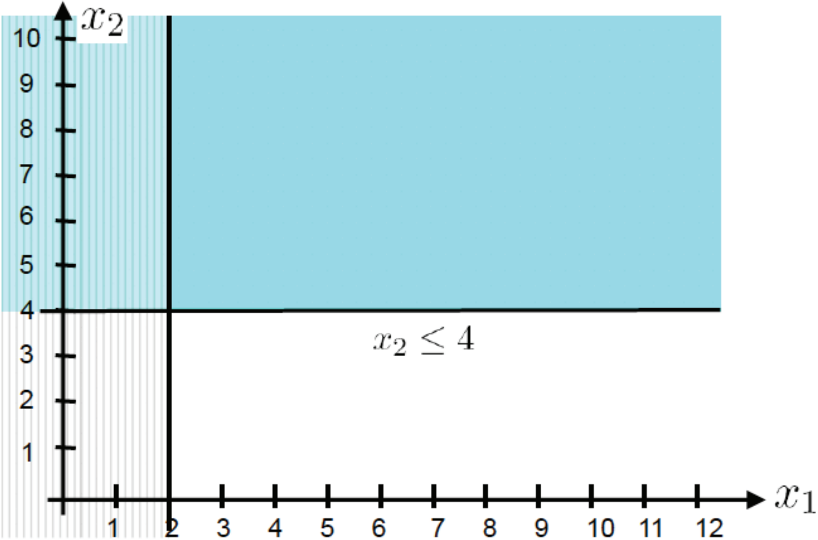

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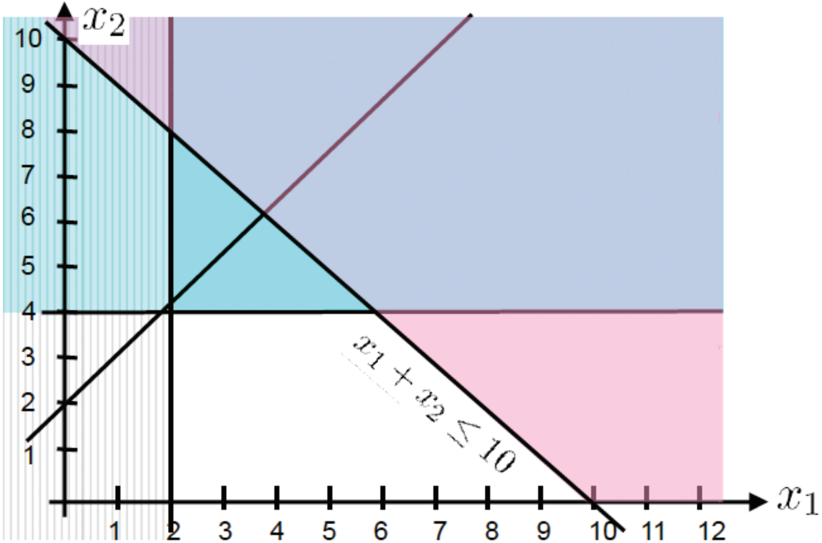

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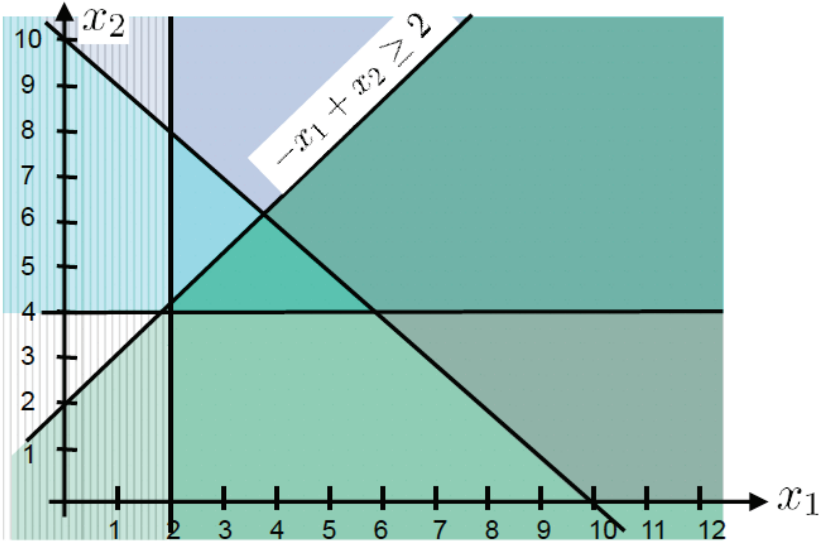
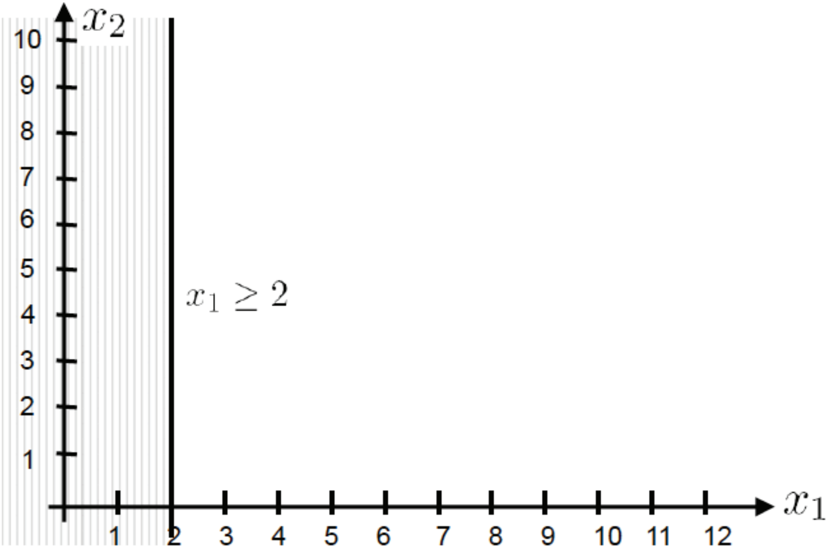
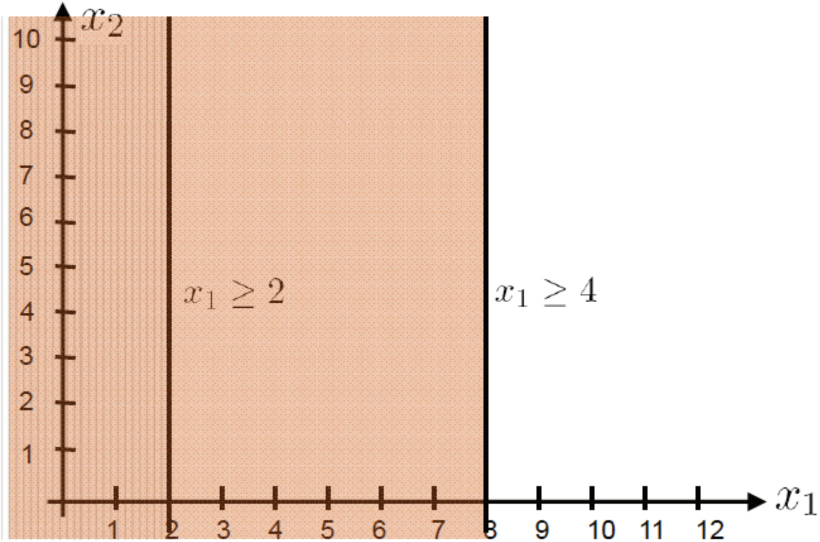


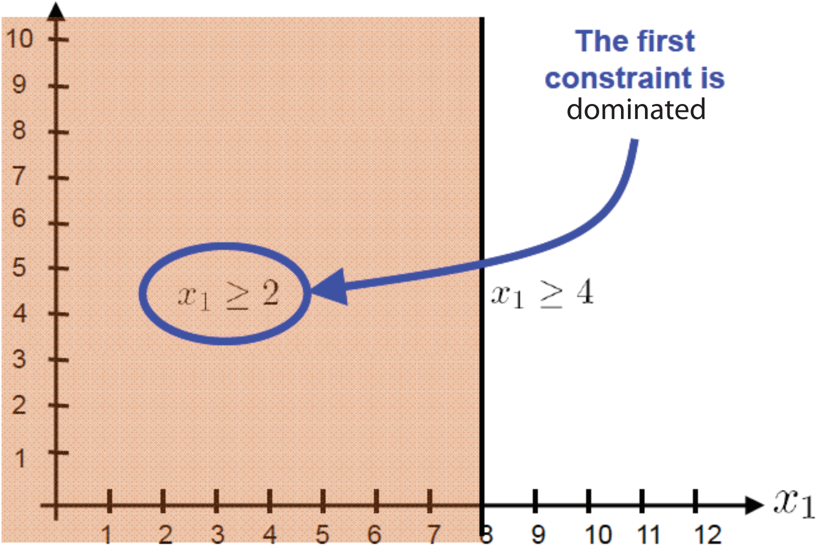
Constraint domination



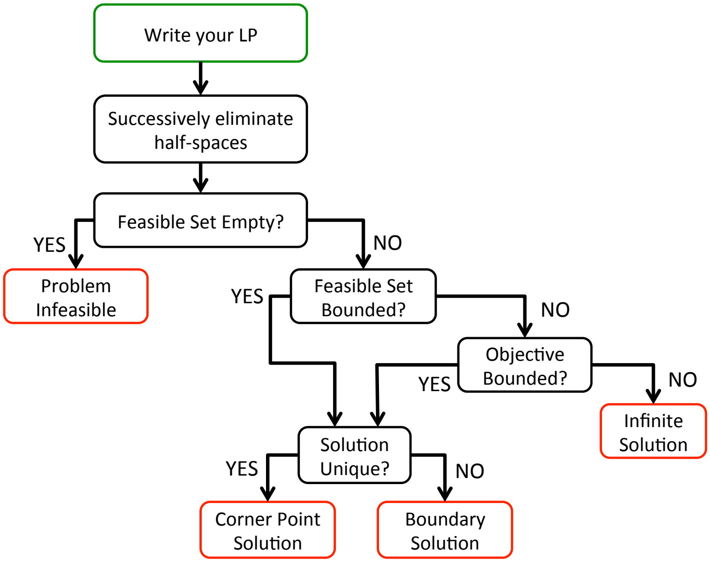
Constraint domination



Constraint domination

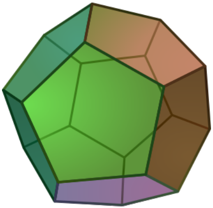


Graphical solution of LPs: A General Method



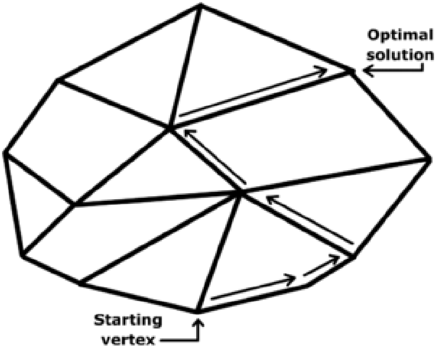
Insights from Graphical LP

- Linear constraints $Ax \leq b$ form feasible set (possibly empty)
- Feasible set is a (possibly unbounded) convex polytope
- Optimal solution exists along edges (corner point or line segment)



Danzig's Simplex Algorithm

1. Define feasible set
2. Start at vertex. Move along vertices until obj. fcn. stops decreasing



Example of Simplex Algorithm

Recall the LP problem:

$$
\begin{aligned}
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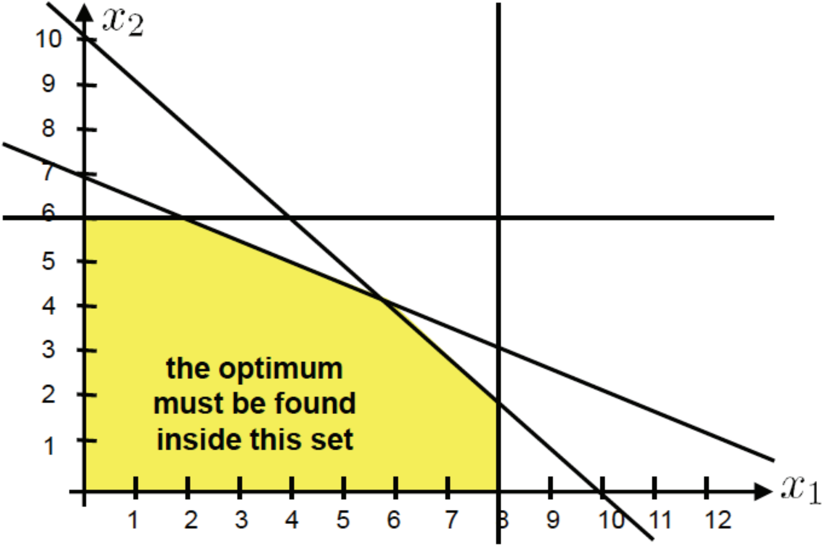
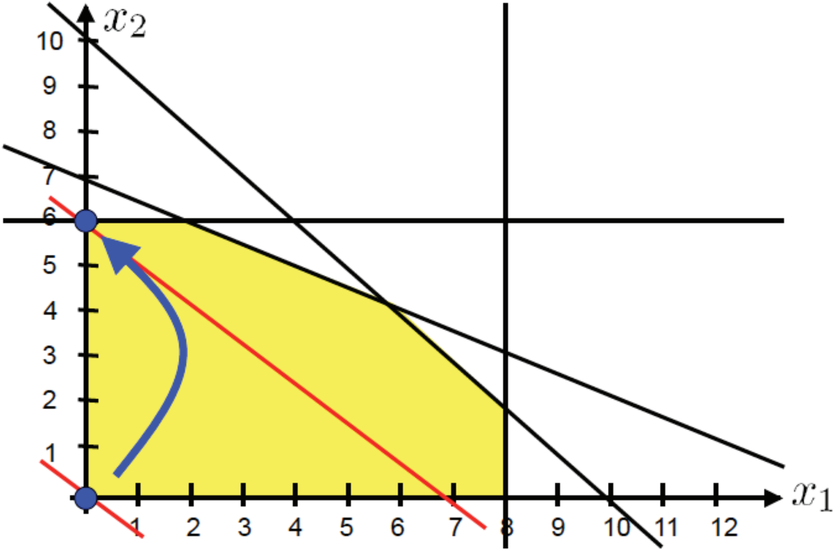
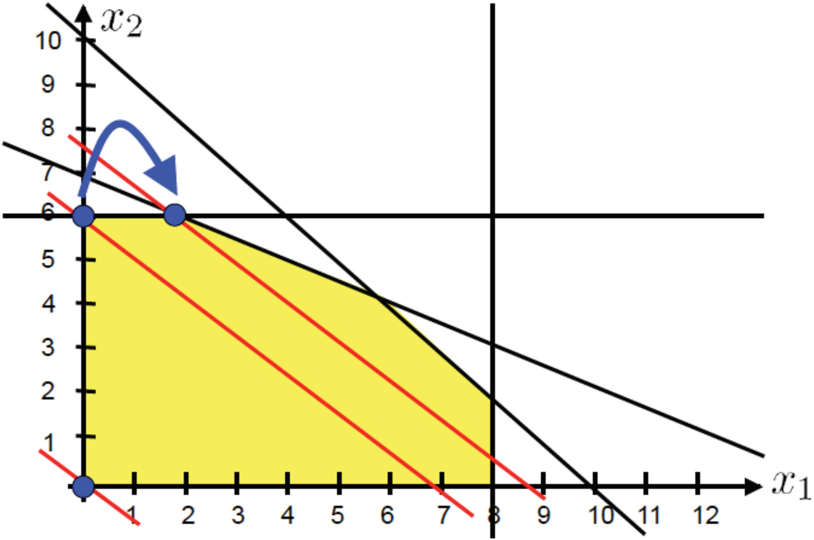


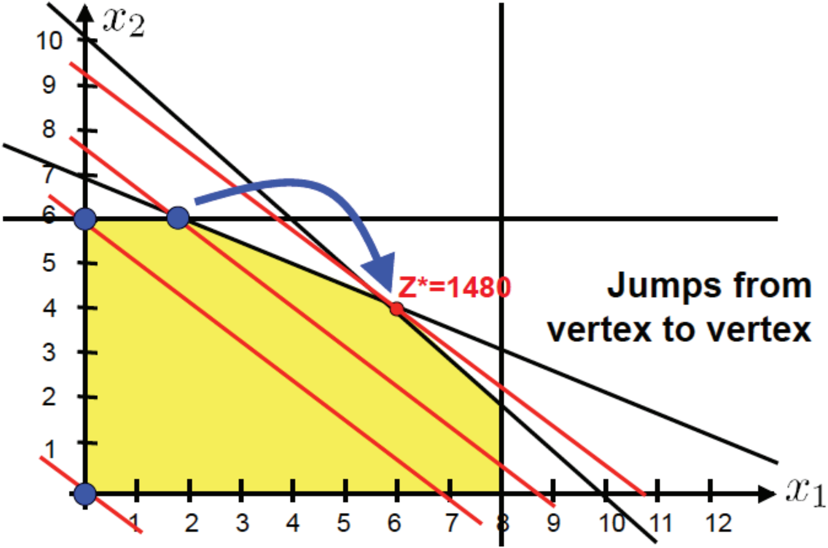
Start at a Vertex



Jump to adjacent vertex



Stop when objective stops decreasing



Jumps from vertex to vertex

Additional Reading

Revelle

- Chapter 3 - A Graphical Solution Procedure and Further Examples

Simplex Algorithm

- Revelle Chapter 4 - The Simplex Algorithm for Solving Linear Programs