CE 191: Civil and Environmental Engineering Systems Analysis

LEC 06 : Integer Programming

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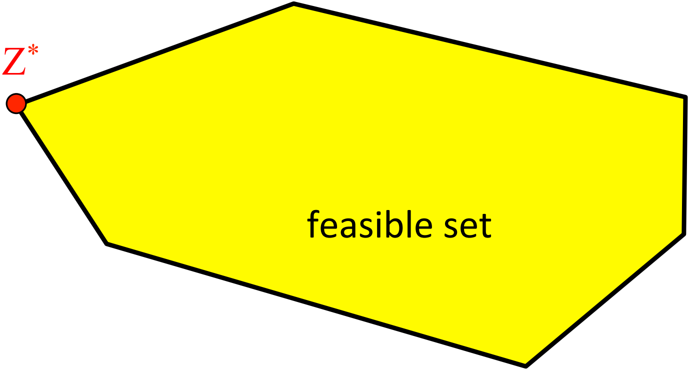
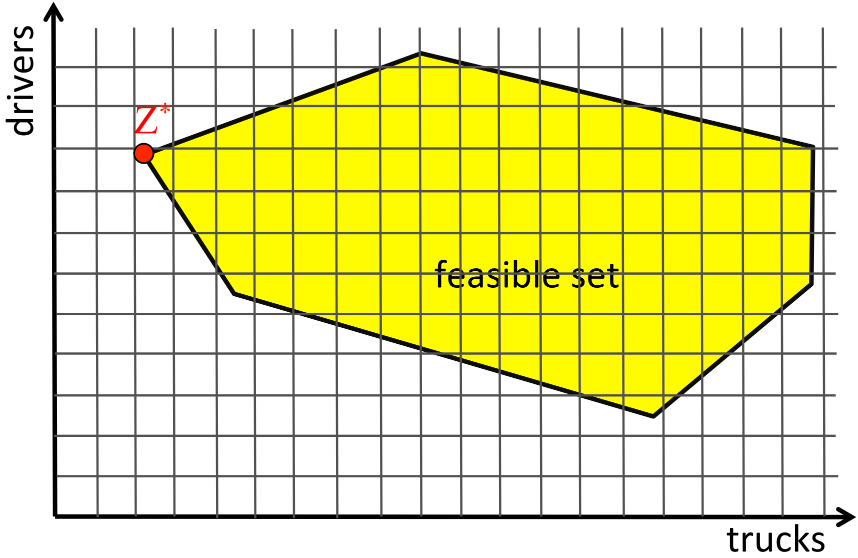
Optimum Z^* of a LP



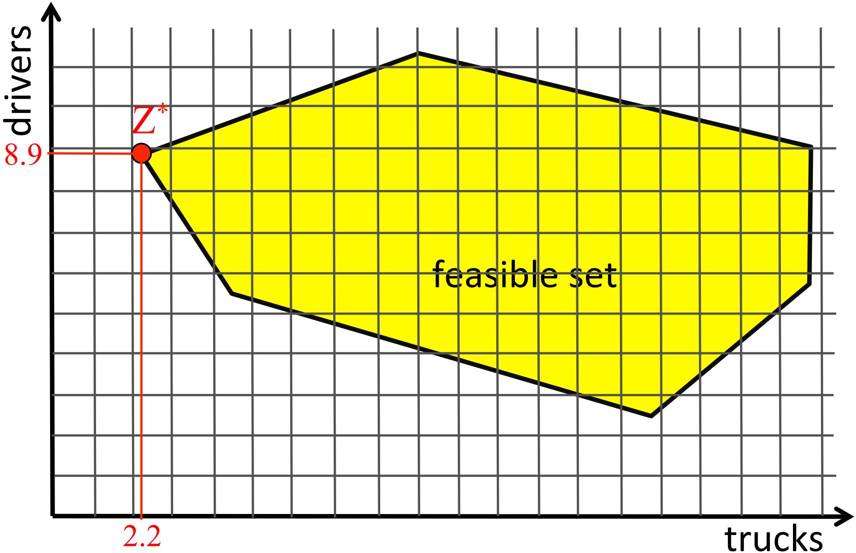
What if the desired solution is an integer?

Suppose the decision variables represent the number of trucks and drivers.



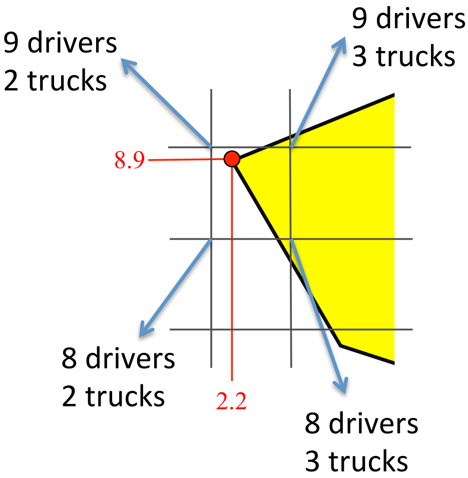
Fractional solution

Suppose the decision variables represent the number of trucks and drivers.



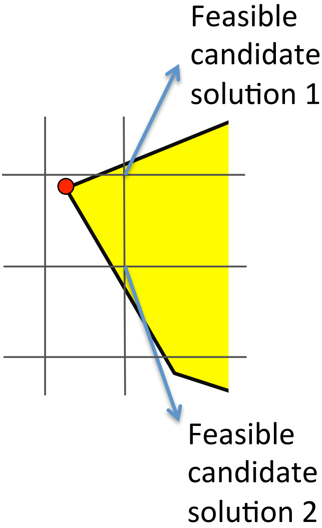
Fractional solution

What should one do?



Fractional solution

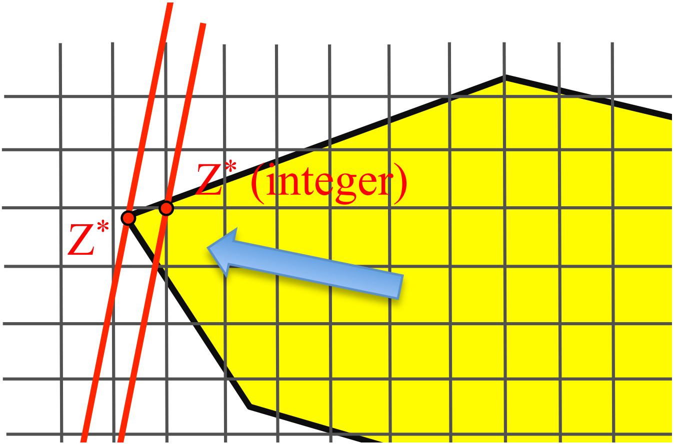
What should one do?



Bounds on the optimum

The fractional solutions provides upper or lower bounds on the optimum.

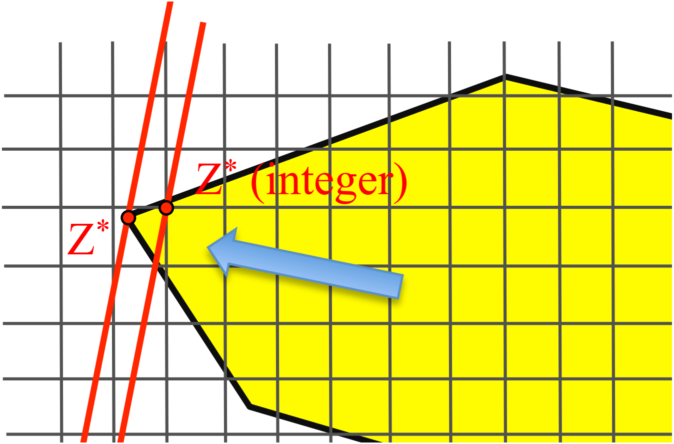
- If a min problem, then it provides a lower bound.
- If a max problem, then it provides a upper bound.



Bounds on the optimum

The fractional solutions provides upper or lower bounds on the optimum.

Integer problems are sometimes very hard to solve exactly. However, sometimes guaranteed bounds on the optimal cost are sufficient (quick & dirty, but correct).



Relaxing Integer problems into fractional problems (LP)

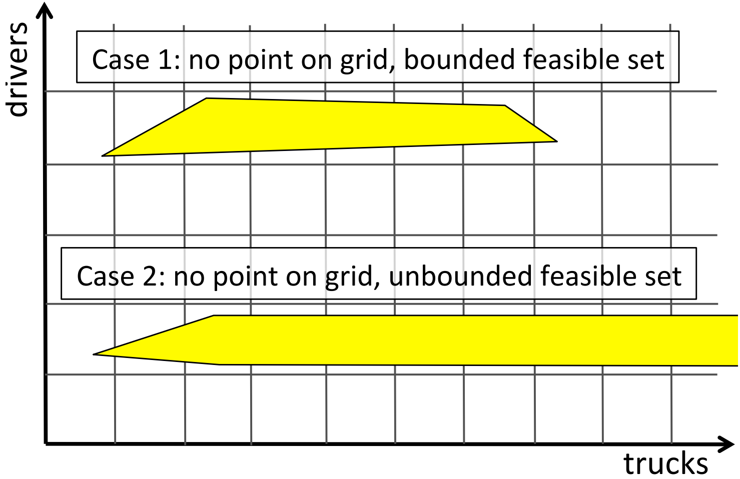
The feasible set for fractional solutions is larger than for integer solutions.

- The result is better, that is
  - If it's a max problem, then the cost is greater
  - If it's a min problem, then the cost is less
- The result may not make physical sense, i.e. 8.9 trucks & 2.2 drivers
- Are there algorithms to determine A / THE optimal integer solution?

  This is a hard problem!

Feasible Sets

Does an integer solution to the problem exist?



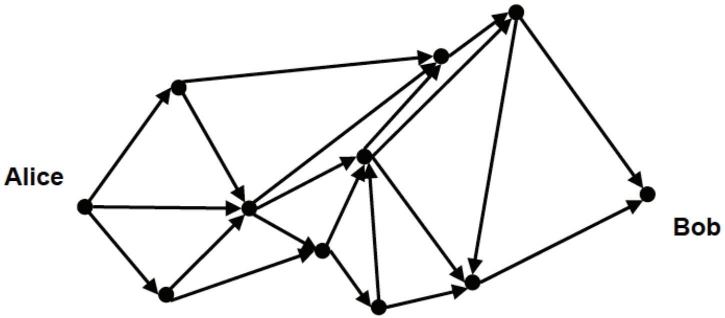
Decision Variables

A possible definition for a decision variable encodes a "yes/no" decision, i.e. a discrete choice.

Example decision variables that can be modeled by integer variables:

1. Do I hire this worker ($x = 1$) or not ($x = 0$)?
2. How many cars are allowed in this parking lot everyday: $x = 0, 1, 2, 3, 4, 5, 6$?
3. Do I take the first ($x = 1$), second ($x = 2$), or fifth train ($x = 5$)?
4. At this intersection, do I take the first left, the second left, the first right?

Shortest Path Revisited: Decision Variables



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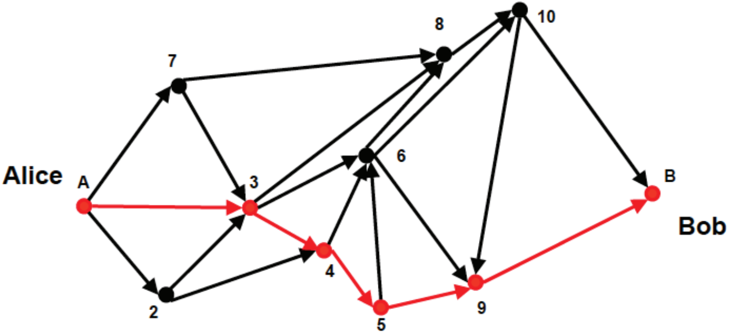

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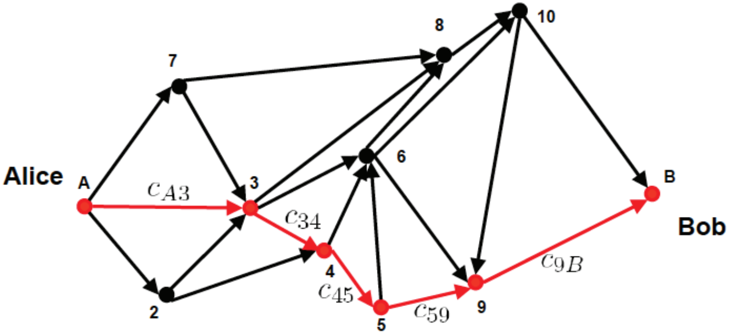
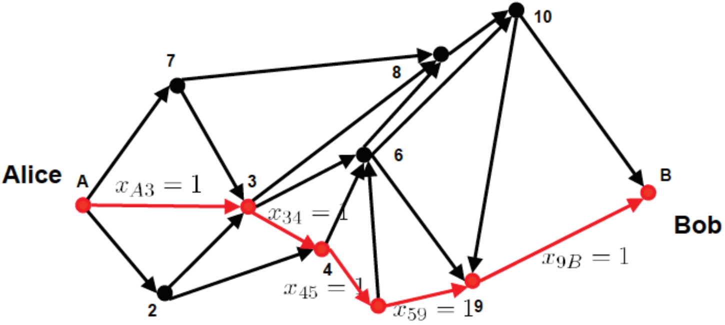


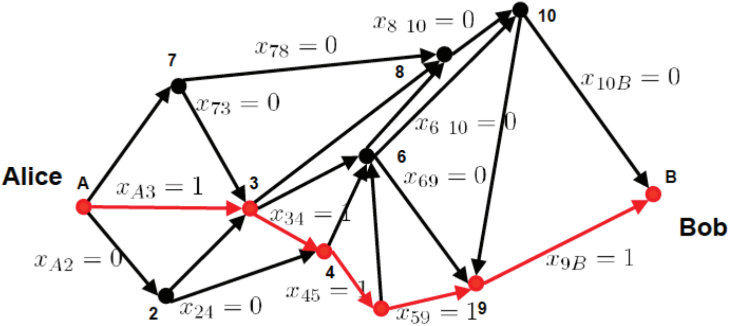
Shortest Path Revisited: Decision Variables



Define

$$x_{ij} = 1 \quad \text{For every } (i, j) \text{ on the shortest path}$$
$$x_{ij} = 0 \quad \text{For every } (i, j) \text{ not on the shortest path}$$

Shortest Path Revisited: Decision Variables



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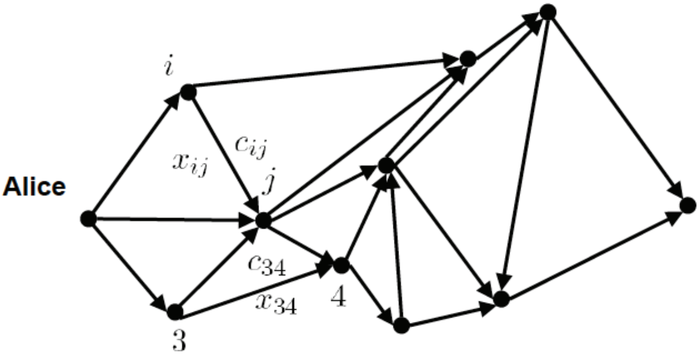

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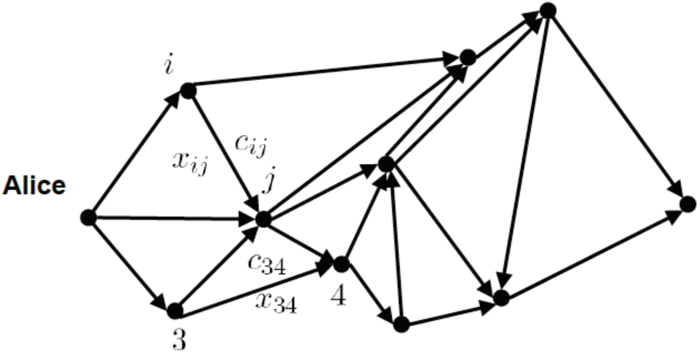
Shortest Path Revisited: Decision Variables

Define a graph (road network)

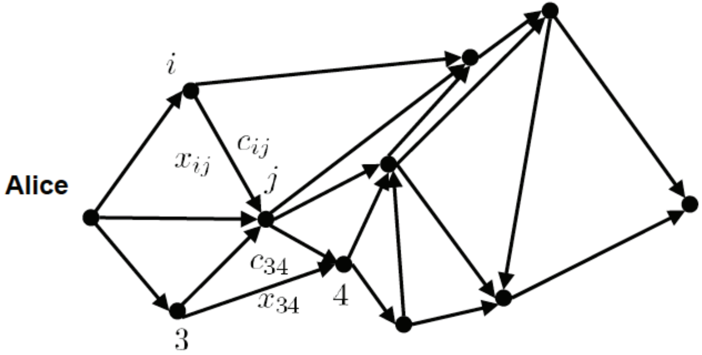


Denote c_{ij} as the cost to go from i to j (e.g. fuel burned)

For example c_{34} is the cost to go from node 3 to node 4

Shortest Path Revisited: Decision Variables

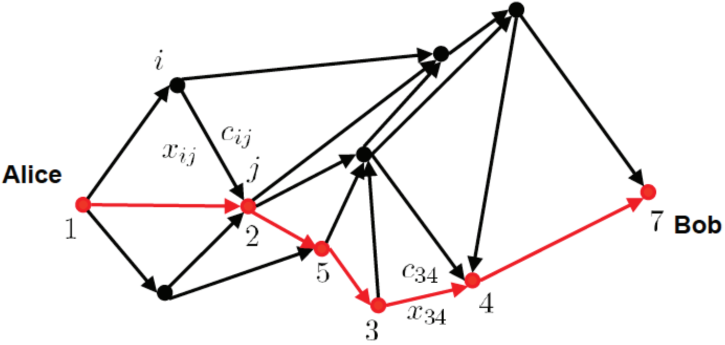
Define a graph (road network)



Take $x_{ij} = 1$ if Alice decides to go through link (i, j), zero otherwise

For example $x_{34} = 1$ if Alice decides to use route (3,4)

Shortest Path Revisited: Decision Variables

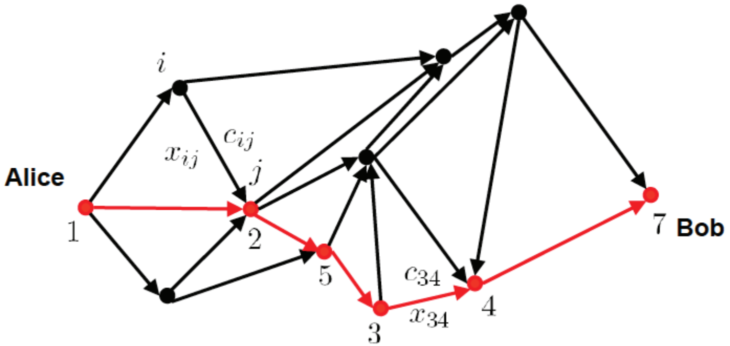


$x_{12} = x_{25} = x_{53} = x_{34} = x_{47} = 1$

All other $x_{ij} = 0$

Total length of this path: $c_{12} + c_{25} + c_{53} + c_{34} + c_{47}$

Shortest Path Revisited: Decision Variables



$x_{12} = x_{25} = x_{53} = x_{34} = x_{47} = 1$

All other $x_{ij} = 0$

Decision variables x_{ij} cannot be fractional, i.e. 0.534.

Q: How do we ensure that the LP provides an integer solution?

A: Dijkstra's algorithm (one possible answer)

Additional Reading

The shortest path problem: Revelle Chapter 6.B