# LEC 16 : Combined Design & Control of a Fuel Cell Bus via Convex Programming

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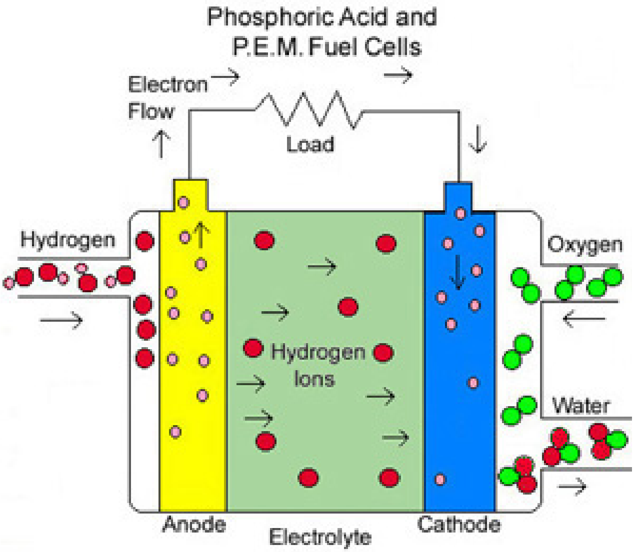
# Fuel Cell Hybrid Vehicles


AC Transit HyRoad Fuel Cell Bus


2016 Toyota Mirai

# How Fuel Cells Work



# Fuel Cell Hybrid Powertrain

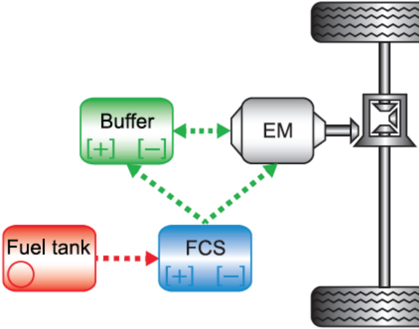



Figure: Fuel cell hybrid powertrain. The vehicle is propelled by an electric machine (EM), which obtains energy from a fuel cell system (FCS), or an electric buffer (battery or supercapacitor). When EM operates as a generator, mechanical energy from the wheels is converted to (and stored as) electrical energy in the buffer.

# Bus Line Velocity and Road Grade

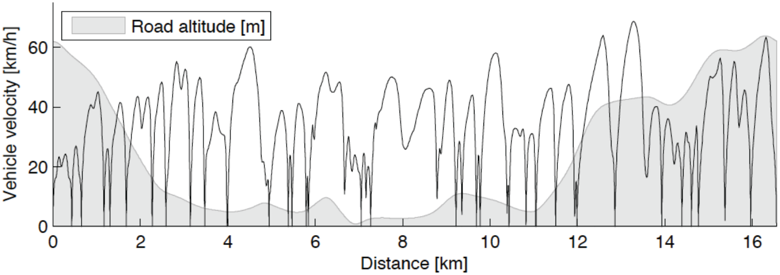



Figure: Model of a bus line, expressed by demanded vehicle velocity and road altitude. The initial and final velocities and road altitudes, respectively, are equal, thus conserving kinetic and potential energy of the vehicle.

## Research Question

Given a fixed bus line (i.e. velocity-road grade profile), optimize

- Fuel cell & super capacitor component sizes
- Energy management strategy for power-split

to minimize operating (hydrogen fuel) + component (FC + SC) costs

## Unique Features

- Component sizes are static design variables (not time-varying)
- Energy management strategy is a multi-stage control process (time-varying)

# Optimization Framework

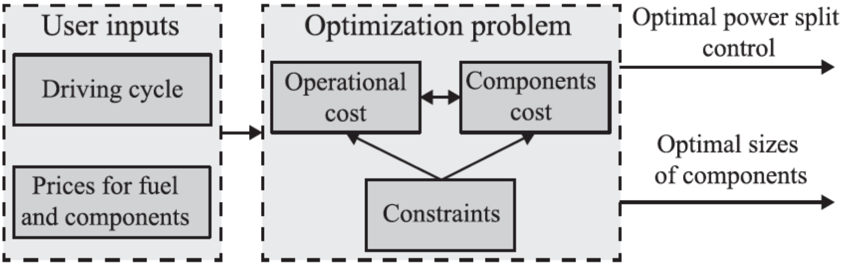



Figure: Optimization framework for simultaneous component sizing and energy management of a hybrid city bus. After user inputs are provided, the combined operational and components cost are minimized simultaneously, in order to obtain the optimal power split control and sizes of powertrain components.

# Optimization Problem

| | |
|---|---|
| minimize | Operation + Component Cost |
| subject to: | Driving cycle constraints, |
| | Energy conversion and balance constraints, |
| | Buffer dynamics, |
| | Physical limits of components, |
| | ... |
| | *(For all time instances along the bus line).* |

# Useful Properties of Convex Function

- The function $f$ is said to be concave if $-f$ is convex.
- An affine function $h(x) = qx + r$ is both concave and convex.
- A quadratic function $f(x) = qx^2 + px + r$ with domain $f$ $\mathbb{R}$ is convex if $p \geq 0$.
- A quadratic-over-linear function $f(x, y) = x^2/y$ with dom $f = \left\{(x, y) \in \mathbb{R}^2 \mid y > 0\right\}$ is convex.
- The geometric mean $f(x, y) = \sqrt{xy}$ with dom $f = \left\{(x, y) \in \mathbb{R}^2 \mid x, y \geq 0\right\}$ is concave.
- A nonnegative weighted sum $f = \sum w_i f_i$, with $w_i \geq 0$, of convex functions $f_i$, is a convex function.
- A product $f(x, y) = xy$ is generally not a convex function.

# Longitudinal Vehicle Dynamics

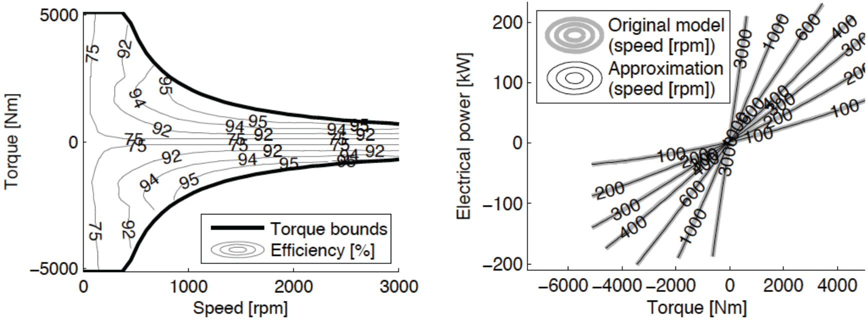
Newton's Second Law, Electric Machine Torque:

$$
\begin{aligned}
T_{dem}(s_F, s_B, t) \quad = \quad & \left( J_V + m(s_F, s_B) \frac{R_w^2}{r_{fg}^2} \right) \dot{\omega}_M(t) + \frac{\rho_{air} A_f c_d R_w^3}{2 r_{fg}^3} \omega_M^2(t) \\
& + m(s_F, s_B) \frac{c_r R_w}{r_{fg}} g \cos \alpha(t) + m(s_F, s_B) \frac{R_w}{r_{fg}} g \sin \alpha(t)
\end{aligned}
$$

Vehicle Mass:

$$
m(s_F, s_B) = m_0 + s_F m_F + s_B m_B
$$

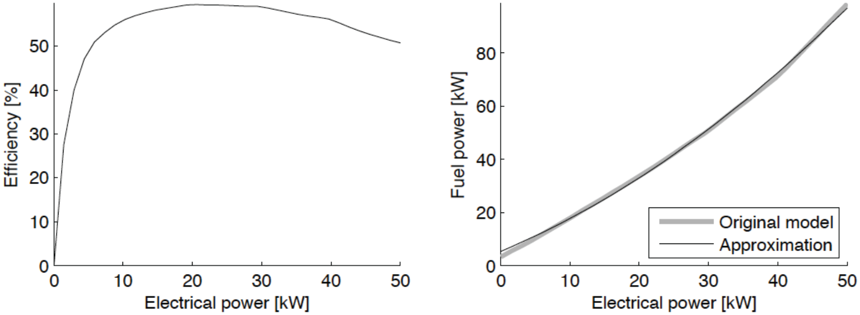
# Electric Motor




(a) Original model. (b) Approximated model.

$$P_M(T_M, t) = a_0(\omega_M) + a_1(\omega_M) \cdot T_M(t) + a_2(\omega_M) \cdot T_M^2(t)$$

# Fuel Cell




(a) Original model.

(b) Approximated model.

$$P_{Ff}(P_{Fe}, s_F) = b_0 s_F + b_1 \cdot P_{Fe}(t) + b_2 \cdot \frac{P_{Fe}^2(t)}{s_F}$$

$$0 \leq P_{Fe}(t) \leq s_F P_{Fe,\max}$$

# Supercapacitor

Energy Storage Dynamics

$$\dot{E}_B(t) = -P_B(t)$$

Resistive Losses

$$P_{B,loss}(P_B(t), E_B(t)) = \frac{RC}{2} \frac{P_B^2(t)}{E_B(t)}$$

The SC energy level $E_B(t)$ is related to the number of SC cells $s_F n_0$ and voltage $V(t)$ according to $E_B(t) = \frac{CV^2(t)}{2} s_B n_0$. Both pack energy, cell voltage, and electric current are limited according to

$$0 \leq E_B(t) \leq \frac{CV_{\max}^2}{2} s_B n_0$$

$$i_{\min} \sqrt{\frac{2n_0}{C} E_B(t) s_B} \leq P_B(t) \leq i_{\max} \sqrt{\frac{2n_0}{C} E_B(t) s_B}$$

KEY STEP: Show these produce convex ineq. constraints, using convex fcn properties

# Objective Function

Minimize operational cost (consumed hydrogen) and component costs (FC and SC)

$$J(T_M(t), P_{Fe}(t), s_F, s_B, E_B(t), P_B(t)) = w_h \int_0^{t_f} P_{Ff}(P_{Fe}(t), s_F) dt + w_F s_F + w_B s_B$$

# Objective Function

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subject to:

- Longitudinal Vehicle Dynamics
- Electric Motor Constraint Equations
- Fuel Cell Constraint Equations
- Supercapacity Constraint Equations

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subject to:

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Must show:

- Obj. Fcn. is convex w.r.t. design variables
- Inequality Fcns are all convex w.r.t. design variables
- Equality Fcns are all affine w.r.t. design variables

Table 5: Pseudo code in CVX for convex optimization of simultaneous component sizing and energy management of a hybrid city bus.

minimize $w_h \sum_{k=1}^{N} \left( b_0 \boldsymbol{s_F} + b_1 \boldsymbol{P_{Fe}}(k) + b_2 \frac{\boldsymbol{P_{Fe}^2}(k)}{\boldsymbol{s_F}} \right) \Delta t + w_F \boldsymbol{s_f} + w_B \boldsymbol{s_B}$

with respect to: $\boldsymbol{P_{Fe}}(k), \boldsymbol{P_B}(k), \boldsymbol{E_B}(k), \boldsymbol{T_M}(k), \boldsymbol{s_F}, \boldsymbol{s_B}, \quad \forall k = 1, ..., N$

subject to:

$$\boldsymbol{T_M}(k) \geq T_0(k) + T_1(k)\boldsymbol{s_F} + T_2(k)\boldsymbol{s_B},$$

$$\boldsymbol{P_{Fe}}(k) + \boldsymbol{P_B}(k) - \frac{RC}{2}\frac{\boldsymbol{P_B^2}(k)}{\boldsymbol{E_B}(k)} - P_a \geq a_0(\omega_M(k)) + a_1(\omega_M(k))\boldsymbol{T_M}(k) + a_2(\omega_M(k))\boldsymbol{T_M^2}(k),$$

$$\boldsymbol{E_B}(k+1) - \boldsymbol{E_B}(k) = -\boldsymbol{P_B}(k)\Delta t,$$

$$\boldsymbol{E_B}(N+1) = \boldsymbol{E_B}(1),$$

$$\boldsymbol{T_M}(k) \geq T_{Mmin}(\omega_M(k)),$$

$$0 \leq \boldsymbol{P_{Fe}}(k) \leq \boldsymbol{s_F} P_{FeBmax},$$

$$0 < \boldsymbol{E_B}(k) \leq \boldsymbol{s_B} \frac{C u_{max}^2 n_0}{2},$$

$$i_{min}\sqrt{\frac{2n_0}{C}\boldsymbol{E_B}(k)\boldsymbol{s_B}} \leq \boldsymbol{P_B}(k) \leq i_{max}\sqrt{\frac{2n_0}{C}\boldsymbol{E_B}(k)\boldsymbol{s_B}},$$

$$\boldsymbol{s_f} > 0,$$

$$\boldsymbol{s_B} > 0,$$

for all $k = 1, ..., N$.

# Optimal Results - I

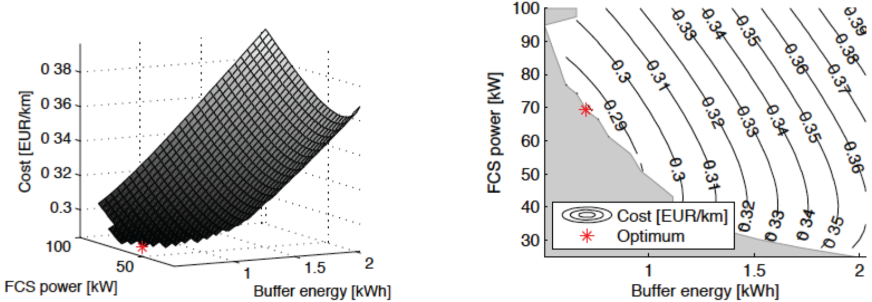
- Optimal results for a FCHV city bus using supercapacitor as an energy buffer:

| Parameter | Value |
|---|---|
| FCS size | 69.3 kW |
| Buffer size | 0.7 kWh |
| Total cost | 0.28 €/km |
| Computational time | <10 s |

Optimal results.

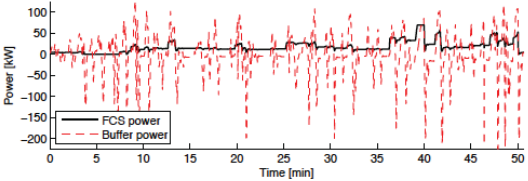
| Parameter | Value |
|---|---|
| Hydrogen price | 4.44 €/kg |
| FCS price | 34.78 €/kWh |
| Supercapacitor price | 10 000 €/kWh |
| Yearly travel distance | 70 000 km |
| Bus' service period | 2 years |
| Yearly interest rate | 5 % |

Prices and bus specifications.

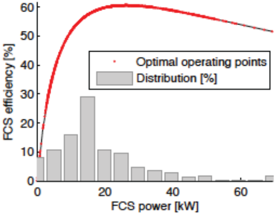



Optimal cost for different sizes of fuel cell system and electric buffer. The shaded region illustrates infeasible component sizes.

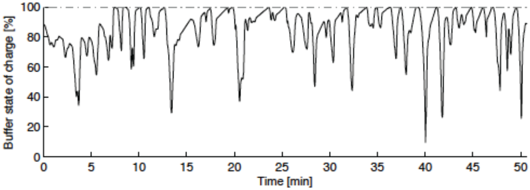
# Optimal Results - II



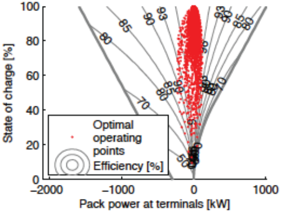

FCS and buffer power trajectories.




FCS's operating points.




Buffer's state of charge trajectory.




Buffer's operating points.

Further details in

[1] N. Murgovski, X. Hu, L. Johannesson, B. Egardt. Combined design and control optimization of hybrid vehicles. Handbook of Clean Energy Systems. Accepted for publication.

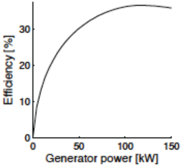
# Summary

- Optimize component sizes (design) and energy management (control) of fuel cell city bus
- Enormously complicated problem
- THE SECRET: convex model formulation
- Solutions in less than 10sec $\rightarrow$ rapid design iteration

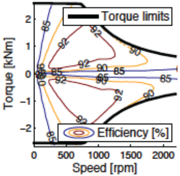
This is just the beginning...

# Case study 3: Plug-in hybrid electric vehicle (PHEV) in a series configuration

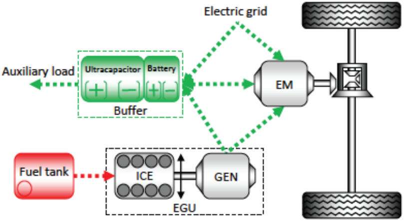
- Dual buffer consisting of Saft VL 45E battery and Maxwell BCAP2000 P270 supercapacitor.
- Can charge at 7 bus stops for 10 s, or 10 min before starting the route.



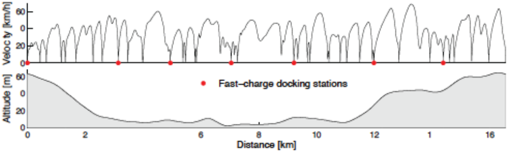

Engine generator unit (EGU).




Electric machine (EM).




Plug-in HEV powertrain in a series configuration.
EGU = Engine generator unit, GEN = Generator.




Driving cycle with charging opportunities.

# Additional Reading


[1]. X. Hu, L. Johannesson, N. Murgovski, B. Egardt. "Longevity-conscious dimensioning and power management of a hybrid energy storage system for a fuel cell hybrid electric bus". *Journal of Applied Energy*, 2014, doi:10.1016/j.apenergy.2014.05.013.

[2]. X. Hu, N. Murgovski, L. Johannesson, and B. Egardt, "Comparison of three electrochemical energy buffers applied to a hybrid bus powertrain with simultaneous optimal sizing and energy management," *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 3, pp. 1193Ð1205, June 2014. doi:10.1109/TITS.2013.2294675

[3]. B. Egardt, N. Murgovski, M. Pourabdollah, L. Johannesson. "Electromobility studies based on convex optimization: design and control issues regarding vehicle electrification". *IEEE Control Systems Magazine*, vol. 34, no. 2, pp. 32-49, 2014, doi:10.1109/MCS.2013.2295709.