Lab 2: Energy Portfolio Optimization

Due: Friday 10/3 at 2:00pm

1 Lab Overview

In this lab, you will learn to formulate a quadratic program to find a mix of energy supplies which minimizes the variability of the price of energy. Section 2 provides a description of the full problem and the constraints. In Section 3, you are asked to form the quadratic program, and implement it in MATLAB. Finally, in Section 4, you will modify the quadratic program to analyze various scenarios. **Please remember to submit your MATLAB code (.m files in one ZIP file), and explain in the report how to run the code.**

2 Energy Portfolio Investment Problem

The California Public Utilities Commission (CPUC) is charged with strategically planning its energy generation over the next several years to meet its growing energy demand. Currently, California generates 199 *megawatt hours* (MWh) from a combination of eight sources. This collection, or mix, of energy sources is known as an *energy portfolio*. The contributions of eight sources to California's current portfolio is listed in Table 1.

Table 1: Current sources of energy to California, and percent contribution to the current energy mix. Source: `http://energyalmanac.ca.gov/electricity/total_system_power.html`

Source	Coal	Hydro	Natural Gas	Nuclear	Biomass	Geo	Solar	Wind
2012 energy mix (%)	7.5	8.3	43.4	9.0	2.3	4.4	0.9	6.3

By the year 2020, California must generate a peak of 225 MWh to meet its growing demand for energy. The expected price in 2020 in dollars per megawatt hour (USD/MWh) is given in Table 2.

Table 2: Expected price of energy in California in the year 2020. Adopted from: Adopted from `http://www.eia.gov/forecasts/aeo/pdf/electricity_generation.pdf`

Source	Coal	Hydro	Natural Gas	Nuclear	Biomass	Geo	Solar	Wind
Expected Price ($/MWh)	100	90	130	108	111	90	144	87

Through technology improvements, California can expand its resources as required to meet future demands. However, the price of each energy source contains uncertainty. Namely, variations in fuel costs, technology

Table 3: Standard deviation of energy source prices in 2020.

Source	Coal	Hydro	Natural Gas	Nuclear	Biomass	Geo	Solar	Wind
σ **($/MWh)**	22	30	15	20	30	36	32	40

development, and future environmental regulations in 2020 impose uncertainty on the expected prices listed in Table 2. Suppose the standard deviation σ of the prices for each source is given in Table 3[1].

The variance in price of an uncertain good (for example 1 MWh of electricity) can be used as a measure of the *risk* of that good. Recall that variance is

$$\text{var} = \sigma^2.$$

Portfolio theory assumes that for a given level of risk, planners prefer lower costs to higher ones. Conversely, for a given expected cost, planners prefer less risk to more risk. By combining various goods in a portfolio, it is possible to create a portfolio with lower risk than any of the goods individually. This is known as *diversification*. The same concepts hold for stock market portfolios.

To ensure competitive electricity rates, California must keep the expected cost of its energy portfolio under 100 USD/MWh. Therefore, California would like to determine an optimal energy portfolio. That is, determine the portfolio with the least risk for the given maximum expected cost.

3 Implementation

1. Formulate a quadratic program (QP) that California can use to minimize the risk of obtaining its energy, while satisfying the maximum expected energy cost constraint described above.
   (a) Define your mathematical notation in a table. Be precise and organized.
   (b) Using this notation, formulate (i) the objective function and (ii) all the constraints.
   (c) Encode this formulation in matrices `Q, R, A, b`, where the QP is formulated as $\min \frac{1}{2}x^T Q x + R^T x$ subject to $Ax \leq b$. Write down matrices `Q, R, A, b` in your report.
   (d) Is the Hessian Q positive definite, negative definite, positive semi-definite, negative semi-definite, or indefinite? Note: the answer would not be trivial if we assumed correlations exist between the standard deviations of energy prices.
2. Solve the QP that you have formulated using MATLAB's `quadprog` command (read the documentation). In your report, provide
   - the value of the objective function, i.e. the risk,
   - the value of the decision variables at the optimum,
   - a qualitative description the optimal solution, including the active constraints.

4 Additional Analysis

The following questions study modifications to the original problem. Each question is independent from the other ones, i.e. the changes are not cumulative. Where applicable, provide (i) the objective function value, and (ii) the optimal decision variable values.

[1] For simplicity, we are ignoring correlations in prices between sources.

3. A concerned and conservative member of the CPUC (yup, they went to Leland StanfUrd JR. University), who is not familiar with portfolio optimization suggests the safest plan is to apply the current energy portfolio mix (Table 1) to the year 2020. **Note:** The mix in Table 1 does not add to 100%. In practice, California imports the remaining energy supply from out-of-state. Apply the same percentages to 2020 as 2012. The total energy supply will not add to 225 MWh. What is the expected risk and expected cost of this portfolio? Note this is a lower bound, because the remaining energy must be imported. Is there a safer (in the sense of less variance) portfolio with at least as small expected cost? Explain.

4. The President of CPUC would like to know the minimal risk of each energy profile ranging in price 50 to 250 USD/MWh. Compute the minimal risk for each energy profile in this price range.

   (a) Summarize your results in a plot with max price [USD/MWh] on the $x-$axis and risk on the $y-$axis. Describe qualitatively the results.

   (b) Describe qualitatively the mix of the lowest risk energy portfolio in this price range. How much does it cost?

   Remark: Ideally, one wishes to both minimize expected cost and minimize risk. This is a multi-objective optimization problem. However, you'll find there exists a tradeoff between these two objectives. To assess multiple objectives we often create the plot above. This is known as *Pareto optimization*[2].

5. By the year 2020, California must generate 225 MWh to meet its growing demand for energy. New information suggests that, due to resource limitations and future governmental regulations, the maximum energy supply of non-renewables sources in 2020 is constrained. The energy supply limits are given in Table 4. Moreover, California has mandated a 33% *renewable portfolio standard* by 2020[3], which includes wind, solar, biomass, and geothermal. Modify the original QP to include these additional constraints.

   (a) Formulate the appropriate constraints imposed by these limits, using the notation of Question 1. Describe any additional notation introduced.

   (b) Solve the original QP with these additional constraints. How does the new solution compare with your results from Question 1? Which constraints are active?

Table 4: Energy supply limits in 2020.

Source	Coal	Hydro	Natural Gas	Nuclear	Biomass	Geo	Solar	Wind
Limit (MWh)	40	50	150	35	10	15	200	50

Deliverables

Submit the following on bSpace. Zip your code. Be sure that the function files are named exactly as specified (including spelling and case), and make sure the function declaration is exactly as specified.

`LASTNAME_FIRSTNAME_LAB2.PDF`
`LASTNAME_FIRSTNAME_LAB2.ZIP` which contains your respective Matlab files.

[2] `http://en.wikipedia.org/wiki/Multi-objective_optimization`
[3] `http://www.cpuc.ca.gov/PUC/energy/Renewables/index.htm`