Lab 3: BAM/PFA Construction: Scheduling via MILPs

Due: Friday 10/24 at 2:00pm

Figure 1: New Berkeley Art Museum (BAM) and Pacific Film Archive (PFA) on Oxford and Center streets.

1 Introduction

In this lab you will solve *Mixed Integer Linear Programs* (MILPs) associated with scheduling painters in a construction project. Recall that MILPs are linear programs in which some of the variables are continuous and some are discrete.

2 Painter Scheduling in a Construction Project

Consider the construction of the new Berkeley Art Museum (BAM) and Pacific Film Archive (PFA) on Oxford and Center streets [1]. During construction, N workers must apply N layers of a protective coating to the building (each worker applies one layer). The N layers are required for the building to pass a certification test (i.e. be considered safe for the public). These N layers can be applied by the N workers in any order, but the effectiveness of the treatment depends on the time interval between the application of any two successive layers. In other words, in order to maximize the life expectancy of this treatment, it is best to wait as long as possible between the application of any two layers. You have to supervise the N workers and schedule

[1] See live http://press.bampfa.berkeley.edu/building/

their jobs. You can only afford to rent the machine needed to apply the coating for one day, so all N layers have to be applied within the same day. The workers are only working part time on this project, and each of them has different schedule constraints. The human resources manager gives you the availability of each worker. Worker i is only available at the following times:

$$\text{Availability of worker } i \; = [a_{i,1}, b_{i,1}] \text{ or } [a_{i,2}, b_{i,2}] \text{ or } \cdots \text{ or } [a_{i,n_i}, b_{i,n_i}] \tag{1}$$

where n_i is the number of time intervals for which this worker is available.

Problem 1:
We suppose that $b_{i,j} < a_{i,j+1}$ for any i between 1 and N, and any j between 1 and $n_i - 1$. What does this mathematical condition represent physically?

Your job is to determine a time t_i for each worker i to achieve his/her task. We assume that the task is performed instantaneously. In order for t_i to be compatible with the workers' constraints, we need to make sure that t_i is in one of the intervals $[a_{i,j}, b_{i,j}]$.

Problem 2:
Are the schedule constraints of the problem of the following form?

$$t_i \in [a_{i,1}, b_{i,1}] \cap [a_{i,2}, b_{i,2}] \cap \cdots \cap [a_{i,n_i}, b_{i,n_i}] \tag{2}$$

If this form is incorrect, provide the correct form of the schedule constraints.

Problem 3:
We want to maximize the time between the application of any two layers of protective coating. Let us call T_{paint} this separation. The problem can be expressed as the following optimization problem.

$$\begin{array}{lll} \textbf{Maximize:} & T_{\text{paint}} & \\ \textbf{Subject to:} & t_i \in [a_{i,1}, b_{i,1}] \cup [a_{i,2}, b_{i,2}] \cup \cdots \cup [a_{i,n_i}, b_{i,n_i}] & \forall i \in \{1, \cdots, N\} \\ & |t_i - t_j| \geq T_{\text{paint}} & \forall (i,j) \in \{1, \cdots, N\}^2 \text{ s.t. } i \neq j \end{array} \tag{3}$$

Imagine that a computer program is able to solve problem (3). You have given a set of values $N = 5$, $a_{i,j}$ and $b_{i,j}$ to the program, and the program returns $t_1 = 10$, $t_2 = 2$, $t_3 = 4$, $t_4 = 14$, $t_5 = 8$. The program does not return the value of the objective function T_{paint}. What is the value of T_{paint} in this example?

Problem 4:
Can problem (3) be expressed easily in a linear manner as seen in class? If it cannot, then list all the "difficulties".

Problem 5:
Your office mate (yup, you guessed it... he's from StanfUrd) apparently found an equivalent way to express the same problem:

$$\begin{array}{lll} \textbf{Maximize:} & T_{\text{paint}} & \\ \textbf{Subject to:} & t_i \geq a_{i,n_i} + \sum_{j=1}^{n_i-1} x_{i,j}(a_{i,j} - a_{i,j+1}) & \forall \; i \in \{1, \cdots, N\} \\ & t_i \leq b_{i,n_i} + \sum_{j=1}^{n_i-1} x_{i,j}(b_{i,j} - b_{i,j+1}) & \forall \; i \in \{1, \cdots, N\} \\ & x_{i,j} \leq x_{i,j+1} & \forall \; i \in \{1, \cdots, N\}, \; \forall \; j \in \{1, \cdots, n_i - 2\} \\ & x_{i,j} \in \{0, 1\} & \forall \; i \in \{1, \cdots, N\}, \; \forall \; j \in \{1, \cdots, n_i - 1\} \\ & t_i - t_j \geq T_{\text{paint}} - M d_{i,j} & \forall \; (i,j) \in \{1, \cdots, N\}^2 \text{ s.t. } i > j \\ & t_i - t_j \leq -T_{\text{paint}} + M(1 - d_{i,j}) & \forall \; (i,j) \in \{1, \cdots, N\}^2 \text{ s.t. } i > j \\ & d_{i,j} \in \{0, 1\} & \forall \; (i,j) \in \{1, \cdots, N\}^2 \text{ s.t. } i > j \end{array} \tag{4}$$

You are not convinced that your office mate is right.

To understand the meaning of $x_{i,j}$, let us first consider an example for worker 2 (i.e. $i = 2$) and $n_2 = 8$: $x_{2,1} = 0$, $x_{2,2} = 0$, $x_{2,3} = 0$, $x_{2,4} = 0$, $x_{2,5} = 1$, $x_{2,6} = 1$. Given all the constraints in problem (4), what is the value of $x_{2,7}$? Given all the constraints in problem (4) and the values of $x_{2,j}$ provided earlier, in which interval does t_2 lie?

Problem 6:
Interpretation of $x_{i,j}$ in the general case: in which situation do we have $x_{i,j} = 0$? In which situation do we have $x_{i,j} = 1$?

Problem 7:
Sketch a simple figure to visually describe the interpretation of x_{ij}. For example, you might draw eight example time intervals and denote x_{ij} for each interval, for worker $i = 2$. You may take a camera phone shot of your hand-sketch and copy & paste it into your report.

3 Computer Implementation

Now that you are convinced that the optimization problem (4) effectively represents your problem, you can put it into the Matlab function `intlinprog` and let it solve the problem.

This section consists of four numerical examples for the optimization problem described in Section 2. Remember that all layers must be applied during the same day (between 5am and 10pm). We also assume that applying protective coating is instantaneous. For example, if a worker is available from 8am to 10am, then we can choose 10am to apply protective coating even though it is the very end of the availability period.

Hint: Before even starting your computer, formulate the optimization problem on paper. Define the decision variables. Define the objective function. Define the constraints. Formulate the LP matrices `c, A, b`. In problems this difficult, this initial step is not just good practice – it is absolutely necessary!!!

Problem 8:
– **Very simple example.**

This first numerical example is very simple and is meant to ensure `intlinprog` produces correct results. You DO NOT NEED to include this example in your report, nor submit the code for it.

There are two workers. Worker 1 is available from 8am to 11am, and worker 2 is available from 1pm to 3pm. At what time should each worker apply a layer of protective coating, so that the time between the application of each layer is maximized?

Problem 9:
– **A slightly more complicated example, but still solvable by hand.**

You can solve this example by hand, and make sure the results given by `intlinprog` are correct. You DO NOT NEED to include this example in your report, nor submit the code for it.

There are three workers. Worker 1 is available from 9am to 12noon, worker 2 is available from 7am to 9pm, and worker 3 is available from 4pm to 6pm. At what time should each worker apply a layer of protective coating?

The following two examples should be included in the report, and the code should be submitted.

Problem 10:
There are eight workers. The availabilities of the workers are the following:

Worker	Availabilities
1	8am-9am, 10am-10:30am, 2pm-2:30pm, 3pm-5pm
2	8am-9am, 9:30am-10:30am, 2pm-3pm, 4pm-5pm, 6pm-7pm, 8pm-9pm
3	7am-8am, 9am-12pm, 3pm-4pm, 5pm-7pm
4	8am-9am, 2pm-3pm, 4pm-5pm, 6pm-7pm
5	8:30am-9:30am, 2pm-2:30pm, 3pm-3:30pm, 5pm-5:30pm
6	8am-9:30am, 2pm-2:30pm, 3pm-4pm, 7pm-7:30pm
7	8am-10am, 10:30am-11am
8	2pm-2:30pm, 3pm-3:30pm, 4pm-5pm

At what time should each worker apply a layer of protective coating?
Hint: In this full version, you should have 61 decision variables (9 continuous + 52 discrete).

Problem 11:
There are eight workers. The availabilities of the workers are the following (identical to the previous problem; except the single alteration in **bold**):

Worker	Availabilities
1	8am-9am, 10am-10:30am, 2pm-2:30pm, 3pm-5pm
2	8am-9am, 9:30am-10:30am, 2pm-3pm, 4pm-5pm, 6pm-7pm, 8pm-9pm
3	7am-8am, 9am-12pm, 3pm-4pm, 5pm-7pm
4	8am-9am, 2pm-3pm, 4pm-5pm, 6pm-7pm
5	8:30am-9:30am, 2pm-2:30pm, 3pm-3:30pm, 5pm-5:30pm
6	8am-9:30am, 2pm-2:30pm, 3pm-4pm, 7pm-7:30pm
7	8am-10am, 10:30am-11am
8	**1pm-1:30pm**, 3pm-3:30pm, 4pm-5pm

At what time should each worker apply a layer of protective coating?

The data for Problems 10 and 11 are very close. Indicate whether the optimal schedules are also close to each other, or not.

Which sentence do you think applies best to the optimization problem in these two examples:

1. The solution of the optimization is robust[2] with respect to small changes in the input data.
2. The solution of the optimization is NOT robust with respect to small changes in the input data.

Deliverables

Submit the following on bCourses. Zip your code. Convert the MS WORD report to PDF.

`LASTNAME_FIRSTNAME_LAB3.PDF`
`LASTNAME_FIRSTNAME_LAB3.ZIP` which contains your respective Matlab files.

[2]A "robust" optimal solution means the solution is still optimal or near optimal when the model parameters are changed slightly.