Lab 4: WIFI for All!

Tower Location Optimization via Nonlinear Programming

Due: Friday 11/07 at 2:00pm

1 Introduction

Imagine Berkeley decides to provide free high-speed WIFI access for all its citizens. You are the lead engineer charged with determining the WIFI tower location and range, as shown in Fig. 1. The goal is to provide maximal coverage for the citizens under budget and geographic constraints.

The problem data is as follows:

- For simplicity, we consider the rectangular section of Berkeley shown in Fig. 1. The coordinates have been scaled such that $(x, y) \in [0, 1] \times [0, 1]$.
- The tower location, to be optimally selected, is denoted by (x_0, y_0).
- The tower's radial coverage, to be optimally designed, is circular with radius R.
- Function $\rho(x, y)$ denotes the population density (e.g. people per square kilometer) at a given location (x, y). Consequently, the number of people with free WIFI is given by

$$f(x_0, y_0, R) = \int_0^{2\pi} \int_0^{R} \rho(x_0 + r\cos\theta, y_0 + r\sin\theta)\ r\ dr\ d\theta \tag{1}$$

  which is the integral of population density over the circular coverage area.
- Due to budget and technology limitations, the maximum radial coverage is limited to $R_{\max} = 0.3$.
- The coverage area is constrained to exist within the grid $(x, y) \in [0, 1] \times [0, 1]$.

2 Start simple. Then increase complexity.

For simplicity, assume $\rho(x, y) = 1\ \forall\ (x, y) \in [0, 1] \times [0, 1]$. That is, the population density is uniform over the entire map.

Problem 1:
Under this assumption, formulate a quadratic program (QP) which maximizes covered citizens under the coverage and budget/technology constraints. Provide the following:

(a) What are the design variables? For ease of notation, group these three variables into vector z.
(b) Derive an objective function to minimize.
(c) Write the constraints in negative-null form (i.e. $g_i(z) \leq 0$) and label their physical meaning.

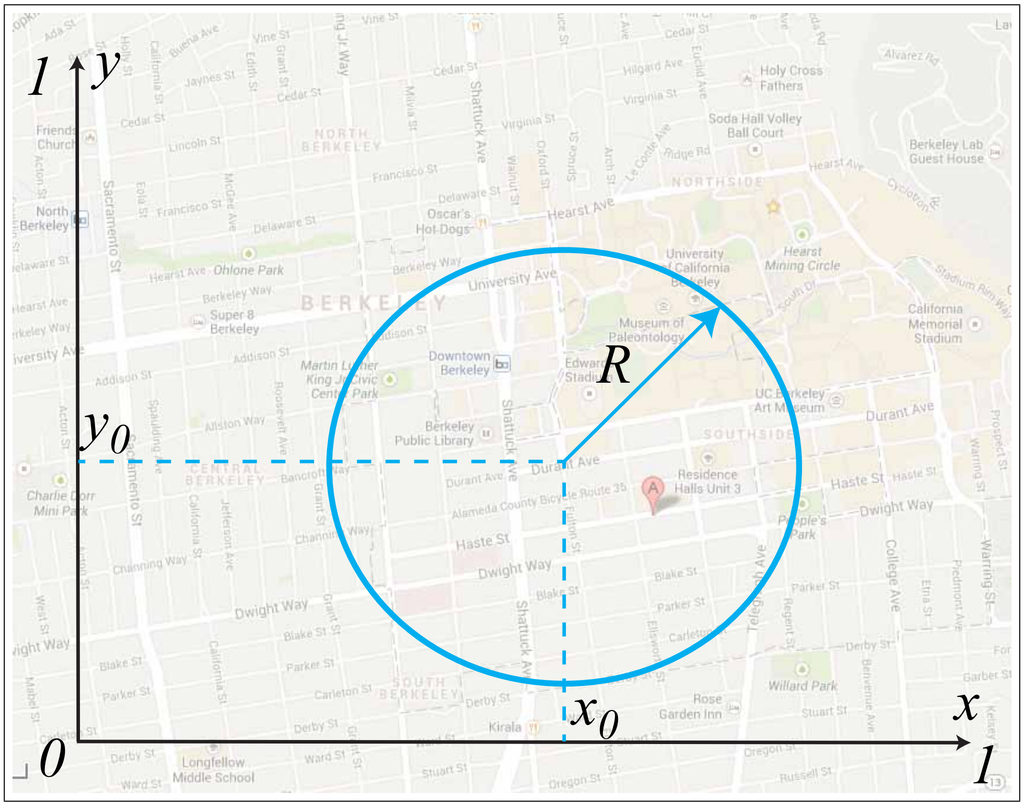

Figure 1: Schematic of optimal WIFI tower location.

(d) Is the feasible set convex? Why? Is the objective function convex over the feasible set? Explain.
(e) Without solving the QP in Matlab, provide the optimal tower location and radial coverage. Explain.

Problem 2:
Now assume a population density $\rho(x, y) = x$. That is, the density increases linearly moving west-to-east, but is uniform north-to-south.

(a) Re-derive the objective function to minimize. Is this problem now a QP? Is the objective function convex? Explain.
(b) Derive the KKT conditions. Make sure to provide all the stationarity, feasibility, regularity, and complementary slackness conditions. You should have 3, 5, 5, and 5 equations/inequalities, respectively.
(c) Without solving the program in Matlab, provide the optimal tower location and radial coverage. Demonstrate this solution satisfies the KKT conditions. What are the values of the Lagrange multipliers?

3 Real Data

Download the file `density.mat` from bSpace. This file contains matrix `rho`, which is a square matrix containing real-world density values, where the rows index variable y, and the columns index variable x. Also download `Lab4.m` and `coverage.m` for skeleton code to solve the subsequent problems.

Problem 3:
Using real-world data for density, compute the objective $f(x_0, y_0, R_{\text{max}})$ for a range $(x_0, y_0) \in [0, 1] \times [0, 1]$ and $R = R_{max}$.

(a) Provide a `surf` plot of the density function $\rho(x, y)$ in your report. Label your axes.
(b) Provide a `surf` plot of the objective function $f(x_0, y_0, R_{\max})$ in your report. Label your axes. Does the objective appear convex, and therefore produce a convex programming problem? Do you expect multiple local maxima? Explain.

4 Nonlinear Optimization

Solve the nonlinear programming problem using `fmincon`. Read the documentation for syntax explanations.

Problem 4:
(a) Provide the optimal tower location (x_0^*, y_0^*), WIFI range R^*, and objective function value f^*.
(b) Which constraints are active? Does this make sense? Provide the non-zero Lagrange multiplier values.
(c) Plug the Geocode into Google Maps. Where should the tower be?

5 Additional Analysis

Problem 5:
Suppose the optimal tower location existed on the Berkeley campus. In this case, the city does not have jurisdiction of the campus and therefore cannot install a WIFI tower in this portion of the map. In other words, the Berkeley campus is an *infeasible set*. Is the feasible set convex? If not, how would one formulate the optimization problem to include this constraint? In words, describe what modifications are required.

Interesting Remarks

- The "density" data is actually Walkscore.com data for Downtown Berkeley, obtained via their API and reverse geocoding.
- This problem sets up the framework to study optimal bicycle sharing station locations.
- This problem is inspired by Project Loon, in which the "tower" is a balloon floating in the stratosphere.
- The function `fmincon` should require a minute or two. The long computation time is due to the solver computing gradients and Hessians via finite differences. In practice, one could fit a polynomial curve to the surface in Problem 3(b). This analytic form makes it possible to analytically derive the gradient and Hessian. Closed-form expressions for the gradient and Hessian can then be provided to `fmincon` via the `GradObj` and `HessFcn` options in `optimset`, significantly reducing computation time.

Deliverables

Submit the following on bCourses. Zip your code. Be sure that the function files are named exactly as specified (including spelling and case), and make sure the function declaration is exactly as specified.

`LASTNAME_FIRSTNAME_LAB4.PDF`
`LASTNAME_FIRSTNAME_LAB4.ZIP` which contains your respective Matlab files.