Bay Area Smart Home Cost Optimization with Photovoltaic Electricity

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Abstract

Climate change will likely be the world's greatest challenge for upcoming generations. California is at the forefront, aiming to address this issue with legislation and incentives but to reach its lofty goals, significant large-scale changes are needed. In recent years, prices for photovoltaic solar panels as well as battery storage have decreased while the technologies continue to improve reaching near market competitive prices. This report investigates smart home energy systems with photovoltaics and battery storage to see how linear program optimization can minimize electricity costs such that the payback times on these technologies are reduced. This article also looks at the sensitivity to fluctuations in the power supplied from photovoltaics and power demanded by the home to see if the LP optimization of averaged historical data could be a viable cost saving solution for smart home energy management. For the smart home detailed in this paper, fluctuations have significant effects on cost over small timescales, but these fluctuations drop off significantly and the mean converges after only a few months. Therefore on the time scale of the payback time of these technologies, the photovoltaic power and power demand fluctuations have insignificant effect on the electricity cost savings.

Introduction

Motivation/Background

Smart homes outfitted with photovoltaic (PV) solar panels and a means of energy storage are able to provide a number of benefits to the power grid and to the homeowner. At the grid level, widely implemented energy storage systems can potentially collectively offset peak power demands and reduce grid stresses that arise from large power demands. Photovoltaic electricity implemented in homes can provide an alternative energy source, which is able to displace local power demands, reducing the total required energy production. Decentralization adds the benefit of redundancy and resiliency in the grid to changes in inputs, demands, and failures. At the home level, benefits are primarily economic: storage allows some decoupling of power demand and electricity withdrawal from the grid, allowing for purchase of electricity at lower rates, whereas PV electricity provides electricity at no cost outside of installation and maintenance.

Also, from an environmental sustainability perspective, PV electricity is able to displace electricity from fossil fuel sources, resulting in carbon savings, and energy storage is able to redistribute the solar energy to demands in different parts of the day to displace even more fossil fuel electricity (if solar input is greater than demand at any point in time). The simple addition of solar panels and a battery, however, are not enough to produce all the listed benefits. Optimization of the system must be done to maximize the utility of the system.

Relevant Literature

Numerous studies have explored optimization of PV and battery equipped smart homes using various techniques. Wang et al., 2012 used mixed-integer nonlinear programming to optimize both cost and user comfort, allowing for flexibility in demands in accordance with proposed user comfort levels [1]. Zhou et al., 2013, in perhaps the most comprehensive study, developed a real-time control strategy to both optimize energy flows and demand scheduling with real-time updates of electricity prices, energy inputs from solar, and uncontrollable demands [2]. Tischer and Verbic, 2006 and Fuselli et al., 2012 examined dynamic programming for optimization of energy flows [3,4].

Focus of Study

In this study we examine a smart home system in Davis, CA outfitted with a photovoltaic cell array, a second-life car battery, and a plug-in hybrid electric vehicle (PHEV) for energy generation and storage. It currently runs an operation scheme based on discretely divided states, which we believe to be suboptimal, as it does not account for the full continuous range of states available for optimization. We aim to simplify the system and to use linear programming to determine a more optimal day-to-day battery operation scheme. However, as a simplified system, a direct comparison between our model system and the actual system might not be possible, so instead we present our designed system as a model that future smart homes energy management systems may be derived from.

Technical Description

Model Formulation

For our analysis we simplify the smart home system by removing the PHEV as a load and battery and assuming that input PV and demand for a day are known perfectly prior to optimization. Demand is assumed to be inflexible. Such a formulation allows for a simple diagram to characterize the system (see Figure 1).

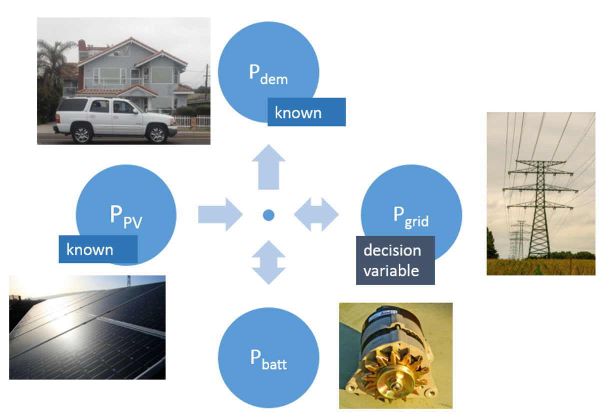

Figure 1: Full PV and Battery System Diagram

A simple energy balance (see Eqn. 2) describes the energy flows in the system at each time step. With two of the variables already known (P_{dem} and P_{PV}), the energy management system only needs to decide one unknown in the time step (P_{grid} in our case) for optimization, as the second (P_{batt}) becomes a dependent variable according to the energy balance equation. A battery dynamics equation (Eqn. 3) links the time steps together, explaining the evolution of the battery's energy level throughout the day as it charges and discharges in accordance to the energy flows at each time step. Battery deterioration is assumed to be negligible. Battery energy level and power we constrain to be within simple limits (Eqns. 5 & 6). As all constraints are linear with respect to our decision variable P_{grid} and the cost function is also linear at each time step, we are able to form a simple linear program (Table 1) to minimize the economic cost day-to-day energy consumption in the home.

Objective Function	$\min_{P_{grid}(k)} [c(k)P_{grid}(k)]$	(1)
Energy Balance	$P_{dem}(k) = P_{PV}(k) + P_{batt}(k) + P_{grid}(k)$ $k = 0, \dots, N$	(2)
Battery Dynamics	$E(k+1) = E(k) - P_{batt}(k)\Delta t$ $k = 0, \dots, N$ $E(0) = E(N) - P_{batt}(k)\Delta t$	(3) (4)
Battery Constraints	$E^{min} \le E(k) \le E^{max}$ $-P_{batt}^{max} \le P(k) \le P_{batt}^{max}$ $k = 0, \dots, N$	(5) (6)

Table 1: LP Formulation

The time step used for this optimization was based on the largest time step of the data we used (1 hour). This optimization could be run for finer temporal resolutions with some minor changes.

Battery/PV Parameters

Our Smart Home optimization is designed around a home equipped with a 10 kWh Lithium Iron Phosphate Battery that has a max power of 5 kW and 12 WIOSUN photovoltaic (PV) panels, 1.3 meters x 1.0 meter each [5, 6]. These parameters and materials were decided to match those of the current model smart home system in Davis, CA.

Cost

To determine the cost of electricity from the grid, we decided to use the "residential Time-of-Use rates" provided by Sacramento Municipal Utility District's (SMUD) option 1 (Table 2), where daily prices are differentiated based on time of day (for on-peak and off-peak periods) as well as season (summer or winter). For our cost function, we assume that we can sell back electricity at the current hour's selling price.

	Summer Season	Winter Season
On-Peak ($/kWh)	0.2420	0.1099
Off-Peak ($/kWh)	0.1130	0.1016

Table 2: SMUD electricity pricing rates

PV/Demand Data

When setting up the energy balance for the linear program, power inputs for both demand and photovoltaics are required for each time step. Since the hourly power demand data is not accessible for the house in Davis, two possible options are to take state-scale data from the California Independent System Operator (CAISO) and scale the hourly averages down to a home-scale level, or to take apartment-scale data from a San Francisco Bay Area, 2 person apartment and scale up to a slightly bigger home. Because CAISO data is grouped together for entire regions, temporal variations at a single residence are smoothed out with millions of other residences.

PG&E's "Green Button" Program allows for homeowners and renter to download their energy-use data on an hourly timescale and so to catch more of these temporal variations, this option was chosen to supply hourly power demands. Because the power demands are determined for a 2 person apartment, to scale the demand to a slightly larger home-scale PG&E's residential customer monthly household energy use average of 538 kWh is used to create a scaling factor for the apartment data [7]. This is done by dividing a household's monthly average energy usage (provided by PG&E) by the average number of days in a month to reach the average daily household demand. This value is then divided by the product of the total of the apartment's hourly demand averages and the time step to acquire a scaling factor (see calculation below).

$$sf_{Dem} = \frac{538\frac{kWh}{month}/30.417\frac{days}{month}}{\Delta t \sum_i P_{dem}} \qquad (7)$$

$$i = 1,2,3, \dots, 24$$

$$t = 1\ hr$$

To get power inputs from PV, solar irradiation data was acquired for Davis, California from the California Irrigation Management Information System (CIMIS). Data was taken over the time period October 1st, 1995 to May 6th, 2014. This extremely large data set (relative to that of the power demand) is useful because of the technique later required for the simulation of solar data (discussed later). The collected data was originally in units of $Langley/day$ and so conversion to units of kW/m^2 is necessary. This value must then be scaled by the total area of the house and the efficiency factor of 18% [8].

$$sf_{PV} = \left(\frac{0.011622\frac{kWh}{m^2}}{1\ Ly}\right)\left(\frac{1\ day}{24\ h}\right)(15m^2)(0.18) \qquad (8)$$

Simulation

From the LP optimization an ideal daily battery power schedule can be found, which is comprised of the power that should be released or stored at each time step in order to minimize the daily cost. For this model, it is assumed power demand and PV power supply (which are based on averages over many days) are known a priori. However, if a LP is used to form the optimal battery schedule a priori, then when variation of power demand and PV power take place on a daily basis, it is important to understand how well the system performs on "non average days". To test the sensitivity of the optimal solution to changes in input data, Monte Carlo simulations were run to add stochastic properties to the input data.

Demands are assumed to be normally distributed about an average demand with a certain standard deviation for each hour, and the demand at each hour is considered independent of the demands of other hours. This models an energy user with randomly varying demand throughout a day and between days, which can be somewhat realistic. The demands are also considered far enough away from zero on average for each hour so that issues of possible negativity in the generated demand can be ignored as unlikely. As such, daily demand data for the Monte Carlo simulation can be generated simply by taking the average hourly distribution (P_{dem}) and adding a vector of randomly generated values from the standard normal distribution, which are first multiplied by the standard deviation at each hour. This is depicted in (9), where x is a randomly generated number with a range (-∞,∞) and a probability distribution ϕ(x).

$$P_{dem,noise} = P_{dem} + \sigma_{P_{dem}} x$$
$$\phi(x) = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \tag{9}$$

For the PV data, the simple method used above in (9) would not be appropriate. The hourly PV values are not normally distributed about the hourly averages (Figure 2) and so using normally distributed random numbers would not model the physical representation of the data.

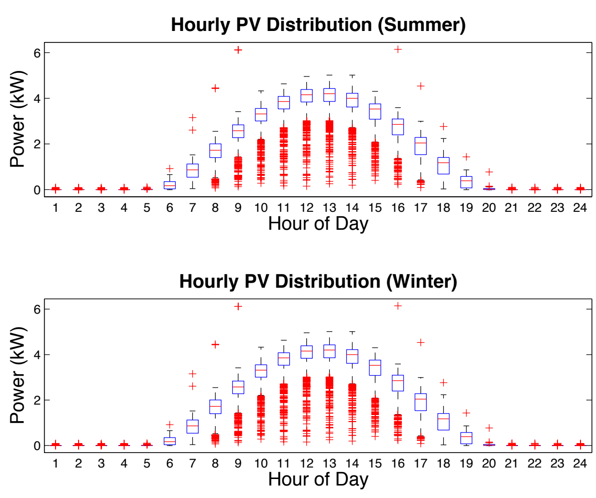

Figure 2: Non-normal PV distributions

Hourly PV values also are correlated to previous time steps however this is assumed to be less true for power demand (i.e. having the lights on at a specific time tells little about whether they will be on in an hour, whereas having the sun at full strength says a lot about the solar irradiation an hour later). To take into account the correlation with previous time steps while bringing randomness into the PV input data, a discrete-time Markov chain is implemented.

$$p_{ijk} = Pr[S_k = j \mid S_{k-1} = i, k-1] \tag{10}$$
$$S_k \in \{S^{min}, S^{min}+1, \ldots, S^{max}-1, S^{max}\}$$

The set up of this Markov chain was based on 24 distinct transition matrices, each one representing a different time, k, throughout the day. To fill each of the transition matrices, hourly solar irradiation data from October 1st, 1995 through May 6th, 2014 was collected. Each hour's transition matrix (of which there were 24 for each season) had rows corresponding to all possible initial solar irradiation levels and columns corresponding to all possible final solar irradiation levels, where initial refers to the beginning of a time step and final refers to the end of the time step (or equivalently the next time step). For example, the first row would have all of the probabilities of going from the minimum solar irradiation level (row 1) at time step $k-1$ to each possible irradiation level at time step k. Each row and each column correspond to a different solar irradiation level with each range spanning the minimum value in the entire data set to the maximum value in the entire data set such that an row index $i = n$ and a column index $j = n$ refer to the same solar irradiation value.

To fill the transition matrices with probabilities coming from data, it first required counts or tallies of each transition that took place. This was done by looping through each hour for each day and looking at the previous solar irradiation value as well as the value for the current hour and then increasing the value by 1 in the corresponding cell of the corresponding transition matrix (each matrix was for a different hour of the day). Once every transition for the 19-year time period had been counted and input in the corresponding place in the set of matrices, the rows of each matrix were divided by the total count of each corresponding row to create a matrix of probabilities (also known as the transition matrix) with each row being a probability mass function (PMF).

These transition matrices are then used to create the cumulative distribution functions for each row of each matrix from the PMF.

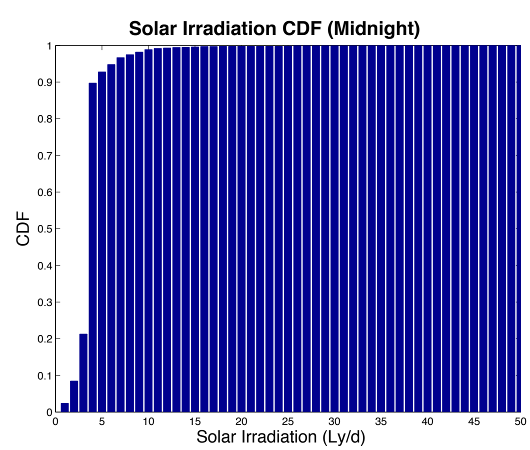

Figure 3: CDF of Solar Irradiation data for 12am

Using the CDFs and a uniform random number generator in the range 0 to 1, Monte Carlo simulations can be run for the solar data. Each time step has an initial irradiation level, and a uniformly distributed pseudorandom number that has been generated in the range from 0 to 1.

For the initial irradiation level, the pseudo random number is mapped onto the CDF to find the corresponding j index (and associated solar irradiation at the next time step). Since the CDF is of a discrete random variable, the pseudorandom number likely does not correspond to an indexed solar irradiation value, S_k, and so linear interpolation is used to calculate the corresponding solar irradiation at the next time.

Once S_k is found it is used as S_{k-1} for the next time step. However because this is from a discrete CDF, the value of S_{k-1} likely does not have a corresponding i index and so the nearest S_{k-1} value and corresponding index, i, are taken to start the next time step. This is repeated for 24 hours and then these values are scaled (by a previously mentioned scaling factor, sf_{PV}) to represent the photovoltaic power supply for a random day.

Now with semi-randomly generated values for PV power supply and home power demand, the optimized battery energy flows are combined in an energy balance to find the grid power for each hour of the day.

$$P_{grid}^{sim} = P_{dem,noise} - P_{PV,noise} - P_{batt}^{*} \qquad (11)$$

where:

$P_{dem,noise}$ is the simulated power demand levels

$P_{PV,noise}$ is the simulated PV power levels

P_{batt}^{*} is the previously optimized battery power inputs/outputs

P_{grid}^{sim} is the grid power for the simulation

**Note: all of these terms are 24 element vectors

For each hour of the day electricity costs are calculated based on grid power and then summed to get a daily energy cost. This simulation is run for an entire season to see how in the long run it fares in comparison to the optimal scenario.

<u>Special Case: Pseudo-Off-the-Grid Approach</u>

In this mode of operation, we assign the sellback price of electricity to be $0 to disincentivize use of the battery for the purpose of buying and selling electricity for profit. To account for this, we separate grid power into two parts ($P_{grid,in}$ and $P_{grid,out}$) with non-negativity equations (Eqn. 18) and reformulate our LP (Table 3). In this way, any withdrawal of electricity is a cost to the system without the potential to earn money by selling the electricity back at a higher price. This results in a system that seeks to minimize energy transfers to and from the grid, thus tending toward an off-the-grid system. We explore first whether or not going completely off-the-grid is possible by running the optimization with the adjusted cost function, and if not, we seek to determine what size of a solar panel may be necessary to put the system effectively off-the-grid.

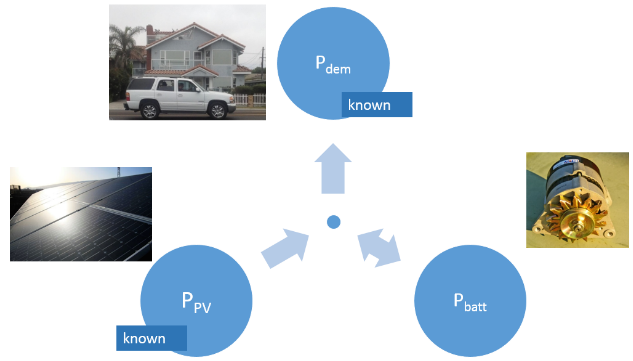

Figure 5: "Off the grid" PV and Battery System Diagram

Objective Function	$\min_{P_{grid,in}(k),P_{grid,out}(k)} [c(k)P_{grid,in}(k) - c(k)P_{grid,out}(k)]$	(12)
Energy Balance	$P_{dem}(k) = P_{PV}(k) + P_{batt}(k) + P_{grid,in}(k) - P_{grid,out}(k)$ $k = 0, \ldots, N$	(13)
Battery Dynamics	$E(k+1) = E(k) - P_{batt}(k)\Delta t$ $k = 0, \ldots, N$ $E(0) = E(N) - P_{batt}(k)\Delta t$	(14) (15)
Battery Constraints	$E^{min} \leq E(k) \leq E^{max}$ $-P_{batt}^{max} \leq P(k) \leq P_{batt}^{max}$ $k = 0, \ldots, N$	(16) (17)
Grid Power Constraints	$P_{grid} = P_{grid,in} - P_{grid,out}$ $P_{grid,in}(k) \geq P_{grid,in}^{min} = 0$ $P_{grid,out}(k) \geq P_{grid,out}^{min} = 0$ $k = 0, \ldots, N$	(18) (19) (20)

Table 3: Modified Off the Grid Formulation

Discussion

Although adding photovoltaics and a battery system to a home can have beneficial effects on both the grid and the environment, commonly the deciding factor for whether or not a system like this gets implemented by a potential user is cost. Therefore, looking at a comparison between the PV-Battery smart home system as described in this paper and a base case where there is neither PV on the roof nor a battery (see Figure 6 below) and estimating the savings, can help determine whether or not these systems are good investments from a financial standpoint. Strong financial benefits would likely be needed for these types of systems to be implemented on a larger scale.

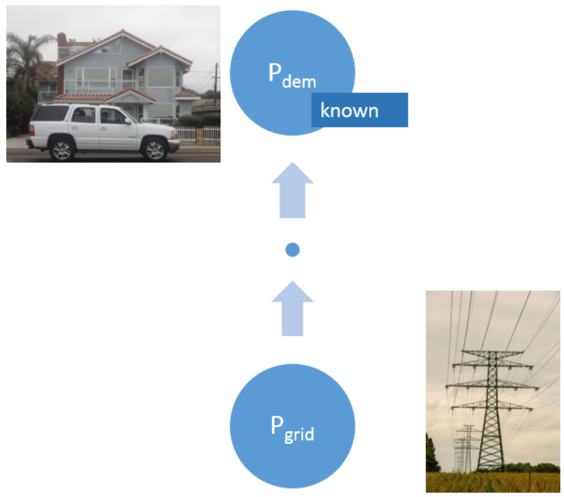

Figure 6: "Base Case" Schematic

As shown in Figure 7 below, the smart home ideally would have an annual savings of about $841 per year. However this is for a case where the power demand and PV power supply are the same every day. We also compared a Monte Carlo simulation run for 122 "summer" days and 243 "winter" days (which were based on the SMUD's seasonal electricity pricing setup where June 1st to September 30th is considered summer and October 1st to May 31st is considered winter). For a single simulation of 365 independent days, we found the saving to be $826, which is still a very significant amount.

	"Base Case"	On Grid, PV+Batt (Optimized)	On Grid, PV+Batt (Monte Carlo)
Summer (avg. daily cost)	$2.78	-$1.68	-$1.64
Winter (avg. daily cost)	$1.84	$0.62	$0.66
Annual Cost	$786.28	-$54.30	-$39.70
Annual Savings	-	$840.58	$825.98

Table 4: Daily and Annual Cost Comparison

To calculate a payback period, we use the assumption that the 10 kWh battery is a 2nd life battery (which are repurposed PHEV or EV batteries that no longer meet vehicle use standards). The associated up front cost is $120/kWh totaling $1200 [5]. We also assume that the cost per peak watt is $3.8 [9]. Converting from cost per peak watt to cost per meter squared (using formula 14), the cost becomes $684/square meter. Using this information and the size of the PV system, the total cost can be calculated to be $10260 for PV leading to a total combined capital cost of $11460. Assuming no maintenance is required over the period of interest, a payback period of slightly less than 14 years is determined. This 14-year payback period is significantly less than many homeownership periods and so if the upfront costs can be addressed it could be a large money saver (and even money maker) for homeowners.

When running these simulations over a large number of days, we found that the mean daily cost (or savings) was very close to the optimal cost (or savings). However, the variability in the daily electricity costs were large, being of the same order of magnitude as the daily costs themselves. On a daily time scale, the optimization is very sensitive to variations in input data, so monthly bills could show significant fluctuation, however this seems to have less significance when looking at the timescale of a payback period or even a year.

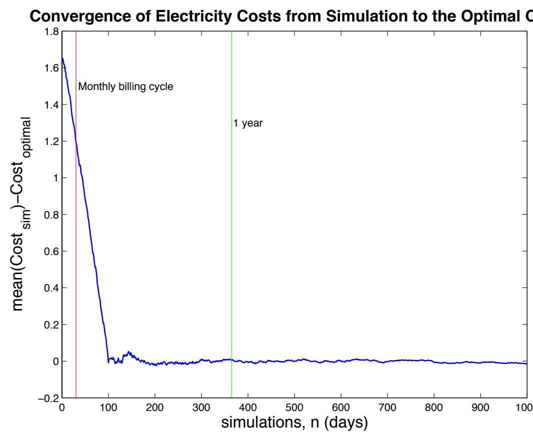

Figure 7: Convergence of Simulation to Optimal Cost

As stated earlier, in terms of environmental sustainability, smart homes equipped with PV and energy storage are able to displace energy produced by fossil fuels, resulting in carbon savings, and able to redistribute the solar energy to demands in different parts of the day to displace more fossil fuel electricity, should if solar power input rise above demand at any point in time.

An interesting case to consider would be to consider what area of photovoltaics panels are needed to effectively go off the grid. The term effectively is used here to mean that over long timescales, the net energy transfer between the grid and the house would be zero. Based on this problem formulation with the previously mentioned parameters the area of PV required to go off the grid would be $26m^2$ with winter being the limiting season.

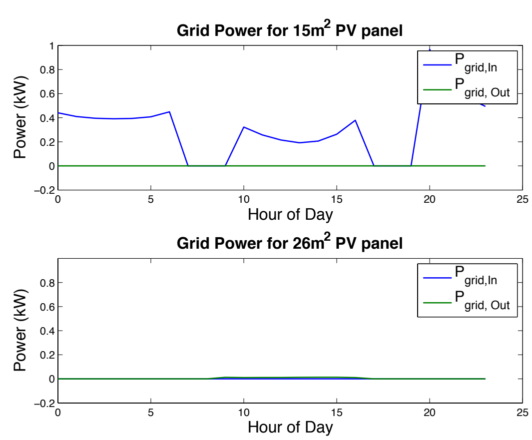

Figure 8: "Off the Grid" external power requirements

Future Direction

Future steps for analysis can include a large number of options. The first would be to increase the complexity of our system to match that of the original system in Davis, CA, accounting for the role of a PHEV as well as adding more constraints and/or costs to account for battery health. This would allow for a direct comparison of the performances of the two-operation schema.

Other options for expansion include the directions that researchers have already covered. This may include allowing for flex loads to affect optimization, accounting for heating and cooling to adjust to ambient temperatures, and exploring dynamic programming and real-time optimization approaches for system operation. Optimization for minimizing carbon emissions, accounting for transmission costs, and performing higher temporal resolution analysis are yet further directions for possible analysis. A more complete model for a system with these considerations may look like Figure 9.

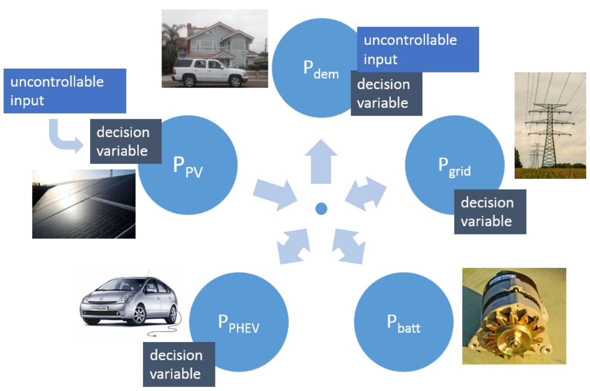

Figure 9: Expanded Model System Diagram

Summary

In summary, we have determined that our simple LP system is able to create significant savings for the smart home owner, >$800 per year for a home with the specifications used in our study. Even with a system programmed to run a single operation schematic for each day for each of summer and winter seasons, over a long time frame, predicted savings can be quite robust despite possibly large day-to-day fluctuations. Many options exist for expanding our model and accounting for other factors to increase accuracy, but we believe our simple model to be a useful base for a simple optimization of smart home energy management systems as well as a useful tool for estimation of the performance of such systems over specified timescales.

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