Optimal Dispatch Model for District Heating Network Based on Interior-Point Method

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***Abstract*—Integrated energy system (IES) and relevant research have been the focus of many scholars in recent years. The coupling between different energy systems including electric, heating and natural gas can be optimized together to improve the overall system efficiency. In this paper, we concentrate on heating system dispatch problem and proposes an optimal dispatch model for district heating network (DHN) with combined heating and power (CHP) units, which considers the thermal balance and hydraulic equations in the pipelines. This is a nonlinear optimization problem which can be solved by interior-point method (IPM). The objective is to minimize the cost of fuel consumption for the whole system. A case study based on 20-nodes DHN with 3 CHP units and 19-branches is demonstrated, proving the effectiveness of the proposed model. Moreover, convergence analysis with comparison solved by traditional IPM and primal-dual IPM is discussed and several possible future research directions are proposed.**

***Keywords*—District heating network (DHN), integrated energy system (IES), optimal dispatch, combined heat and power (CHP), convergence analysis.**

NOMENCLATURE

Parameters:

N Total number of nodes in DHN
B Total number of pipelines/branches in DHN
A Thermal network node-adjacent association matrix, N×B matrix
A^{Exp} Expanded Thermal network node-adjacent association matrix, $2N$×$(2B+N)$ matrix
$\overline{A}$ Expanded up association matrix
$\underline{A}$ Expanded down association matrix
T_a 2B×1 column vector, ambient temperature for pipe branches, unit: ℃
T_{load}^{const} Mass flow temperature of load nodes into return pipe, unit: ℃
T_{res}^{const} Mass flow temperature of resource nodes into supply pipe, unit: ℃
$T_{ns,min}/T_{nr,min}$ Minimum temperature of supply/return network
C_p Heat capacity of mass flow (water), unit: MW/kg·℃
L_i Length of pipe *i*
λ_i Heat conducting coefficient of pipe *i*
$\underline{A}'$ First 2B clomuns of $\underline{A}$
$\emptyset^{load}$ Set of load nodes
$\emptyset^{res}$ Set of resource nodes
$\emptyset^{noload}$ Set of nodes without load and resource
φ^{CHP} Set of CHP units

Variables:

p_i Electric power output of CHP unit *i*
h_i Heat power output of CHP unit *i*
M_s B×1 column vector, mass flow of branches, unit: kg/s
M_q N×1 column vector, inflow of nodes, unit: kg/s
T_s N×1 column vector, supply temperature of node, unit: ℃
T_o N×1 column vector, return temperature of node, unit: ℃
T_{start} 2B×1 column vector, start temperature of pipe branches, unit: ℃
T_{end} 2B×1 column vector, end temperature of pipe branches, unit: ℃
T_{ns} N×1 column vector, node temperature of supply network unit: ℃
T_{nr} N×1 column vector, node temperature of return network unit: ℃
E coefficients of heat loss in each pipe
Q_J N×1 column vector, heat resources/loads of node, unit: MW

I. Introduction

Integrated energy system (IES) [1]-[3], also known as comprehensive energy network [4][5] or multiple energy carriers [6][7], which is one of the basic components of the concept "energy internet" (EI) [1], has been the focus of many researchers these years.

For example, there are already several literature discussing the coupling between bulk electric power system and natural gas system in terms of system planning so far. Ref [8] presents an optimal expansion planning model for an energy hub with multiple energy systems, in which appropriate investment candidates for generating units, transmission lines, natural gas furnaces and CHPs can be determined. Ref [9] proposes an integrated formulation for the steady-state analysis of electricity and natural gas coupled systems. A mixed integer nonlinear programming (MINLP) problem is proposed in [10] to optimize investments and enhance the reliability of coupled gas and electricity networks. However, due to the limitation of expansion planning problem formulation, optimal dispatch of IES has not been considered in these studies.

Besides that, there are also some studies focusing on solving DHN operation problem together with optimal dispatch for electric power system [12]~[15]. Electricity and heat networks were investigated as a whole by combined analysis in [12]. In addition to that, [13], [14] and [15] considered the time-delay character and thermal storage capability within the pipes. It has been proved that wind energy curtailment was significantly reduced in a provincial electric power system by utilizing the thermal energy storage capability of DHN in [13] and [14], with MINLP and MILP respectively. Furthermore, indoor temperature control and building thermal inertia are considered in a multi-regional coordinated operation strategy based on model predictive control (MPC) in [15]. Basically these research are based on some earlier results in the field of thermal engineering analyzing the heating system operation problems[17], which depicts the characteristics of water flow dynamics and heat loss in the pipeline with partial differential equation. Usually the complex model is simplified to steady-state equations without differential part and also proved to be effective. Therefore, MILP model is also applied in related research when the construction of pipes needs to be optimized, i.e. a DHN planning problem is to be solved[16], or when decentralized control is necessary[15]. However, the convergence analysis has not been explored in these literature.

Considering research work in above-mentioned literature, an optimal dispatch model for district heating network (DHN) is proposed and discussed in this paper, with respect to the thermal balance and hydraulic equations in the pipelines. The objective is to optimize energy-supply cost in steady state, without partial differential equations describing the dynamics of the system. We also compare the convergence speed of calculations with different algorithms solving sub-problem in interior-point method, including Broyden–Fletcher–Goldfarb–Shanno (BFGS) and Conjugate Gradient (CG), which can provide some helpful opinions about how to integrate this model into energy system optimization problems between interdependent infrastructures, e.g. electric power grid and natural gas pipelines, so that this model can be utilized in IES optimization problem more efficiently in the future.

The contributions of this paper are summarized as followed:

(1) The DHN is formulated in different perspective in terms of network topology, with heat resources and loads also on branches instead of nodes. In this model, mass flow rate in pipes and CHP units power output are all control variables.

(2) Convergence analysis for this model with comparison between case studies solved by traditional IPM and primal-dual IPM is discussed. Also, several possible future research directions are proposed.

The remainder of this paper is organized as follows: the DHN model formulation is presented in Section II. The result of case studies is demonstrated in Section III, where the model is applied to solve a 20-node and 19-branches DHN, which prove its effectiveness and efficiency. Conclusion and some helpful opinions about future research are given in Section IV.

II. Model Formulation

A DHN optimal dispatch model is introduced in this section. The features of this model are as below:

- CHP units and DHN are modeled, in which mass flow rate, temperature of heat supply and heat power output of CHP units are considered as control variables.
- The objective is to minimize the overall cost of fuel of CHP units.
- Heat losses in the pipelines are considered.
- Heating water supply and return pipelines are model by symmetric topology.
- Heat resources and loads are both characterized as branches in topology of DHN.

The DHN model is illustrated in Figure 1 and described in details in following subsections.

A. *Topology of DHN with resources and loads on branches*

To describe a district heating system with *N* nodes and *B* branches (supply pipes), namely a directed graph, we use node-branch incidence matrix *A* in terms of topology theory, each element of *A* can be expressed as [19][20]:

$$a_{ij} = \begin{cases} 1, & if\ node\ i\ is\ the\ start\ node\ of\ pipe\ j, \\ -1, & if\ node\ i\ is\ the\ end\ \ node\ of\ pipe\ j \\ 0, & if\ node\ i\ is\ not\ related\ to\ pipe\ j \end{cases} \quad (1)$$

Obviously, matrix *A* has *N* rows and *B* columns because of the node and branch number in DHN. For example, in Figure 1, *A* is a 4×3 matrix because there are 4 nodes (1 heat resource, 2 heat loads, 1 transfer node) and 3 supply pipes.

Assuming that the load and resource nodes are also branches in DHN, the expanded node-branch association matrix A^{Exp} is proposed to describe the DHN, with *N* branches added. Besides that, considering return network as completely symmetry of supply network as well as the mass flow rate in the pipes,

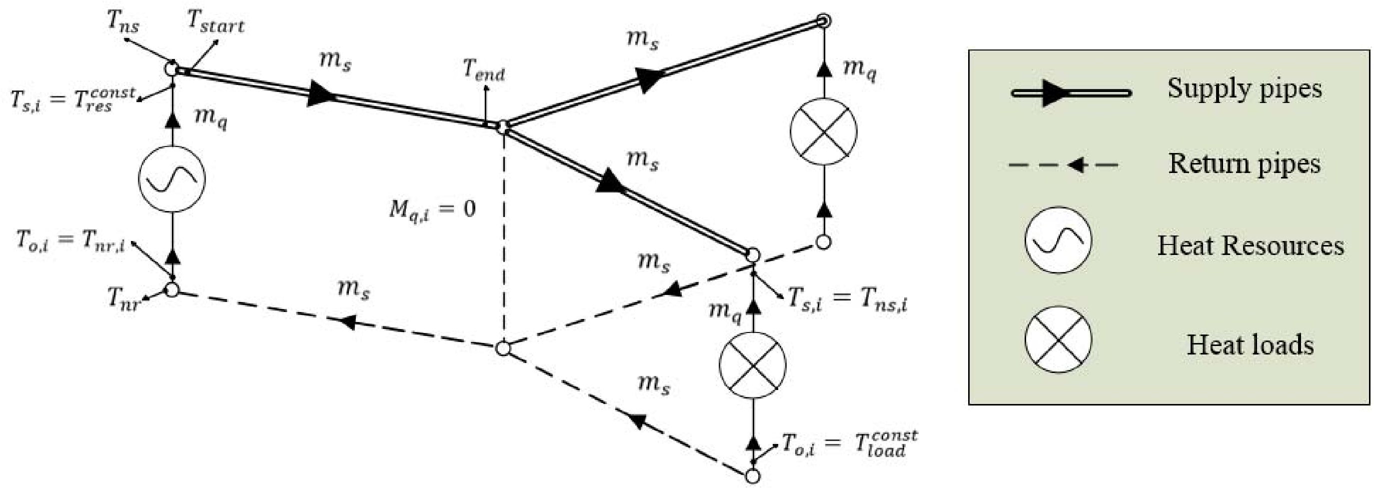

Figure 1 DHN model with heat resources and loads on branches

another B branches are added in the matrix. Therefore, A^{Exp} has $2N$ rows (nodes) and $B+N+B=(2B+N)$ columns (branches)

$$A^{Exp} = \begin{bmatrix} A & 0 & -I \\ 0 & -A & I \end{bmatrix} \tag{2}$$

For example, in Figure 1, A^{Exp} is a 8×10 matrix, because there are 4 rows and 7 columns added in total.

Besides A^{Exp}, there are also up association matrix $\overline{A}$ denotes that if branch j ends at node i, $a_{ij} = 1$, otherwise $a_{ij} = 0$. Similarly, in down association matrix $\underline{A}$, if branch j starts from node i, $a_{ij} = 1$, otherwise $a_{ij} = 0$. It's noteworthy that A, A^{Exp}, $\overline{A}$ and $\underline{A}$ are all known parameters of DHN.

The benefits of this DHN model, with heat resources and loads on branches, mainly relies on the convenience of calculating temperature mixing of mass flows. With this expanded DHN model, mass flows from heat resource and load nodes are the same as mass flows from different pipes, which makes the expression of temperature mixing more intuitive. Also, in this model, supply and return temperature for heat resources and loads (T_s and T_o), start and end temperature of pipes (T_{start} and T_{end}), temperature of node in supply and return network (T_{ns} and T_{nr}) are all control variables. Even though some of these temperature variables are related with each other or equal to some constant value, it still offers strong scalability in terms of temperature control in DHN.

B. Steady state equation of fluid mechanics

The mass flows within the pipes are formulated in constraints below:

$$AM_s = M_q \tag{3}$$

$$-M_{s,max} \le M_s \le M_{s,max} \tag{4}$$

$$-M_{q,max} \le M_{q,i} \le -M_{q,min} \quad , \qquad i \in \emptyset^{load} \tag{5}$$

$$M_{q,min} \le M_{q,i} \quad , \qquad i \in \emptyset^{res} \tag{6}$$

$$M_{q,i} = 0 \quad , \qquad i \in \emptyset^{noload} \tag{7}$$

Eq. (3) denotes the relation between mass flows in pipes and nodes. Eq. (4), (5) and (6) limit the mass flow rate to a reasonable range. Eq. (7) represents that there is no mass flow between supply and return network if this node has no load at all. It's noteworthy that (3) is equal to

$$A^{Exp} \begin{bmatrix} M_s \\ M_s \\ M_q \end{bmatrix} = 0 \tag{8}$$

Which means that the pipes are a closed system without any water injections from outside. However, we apply (3) instead of (8) because there are redundant rows in (8) and it's obvious that (3) is a simplified version.

The electric power consumed by water pump [19][20] to maintain the mass flow rate is not counted in this formulation due to its relatively small capacity according to practical knowledge.

C. Steady state equation of heat conditions

$$Q_J = C_p diag(M_q)(T_s - T_o) \tag{9}$$

$$T_{end} - T_a = diag([E\ E])(T_{start} - T_a) \tag{10}$$

$$E = diag(e^{\frac{-\lambda_1 L_1}{C_p m_1}}, e^{\frac{-\lambda_2 L_2}{C_p m_2}}, \cdots, e^{\frac{-\lambda_B L_B}{C_B m_B}}) \tag{11}$$

$$C_p \overline{A} diag\left(\begin{bmatrix} M_s \\ M_s \\ M_q \end{bmatrix}\right)\begin{bmatrix} T_{end} \\ T_s \end{bmatrix} = C_p \underline{A} diag\left(\begin{bmatrix} M_s \\ M_s \\ M_q \end{bmatrix}\right)\begin{bmatrix} T_{start} \\ T_o \end{bmatrix} \tag{12}$$

$$\underline{A}' \begin{bmatrix} T_{ns} \\ T_{nr} \end{bmatrix} = T_{start} \tag{13}$$

$$T_{s,i} = T_{ns,i} \quad i \in \emptyset^{load} \tag{14}$$

$$T_{o,i} = T_{load}^{const} \ i \in \emptyset^{load} \tag{15}$$

$$T_{o,i} = T_{nr,i} \quad i \in \emptyset^{res} \tag{16}$$

$$T_{s,i} = T_{res}^{const} \ i \in \emptyset^{res} \tag{17}$$

$$T_{ns,min} \leq T_{ns} \leq T_{res}^{const} \tag{18}$$

$$T_{nr,min} \leq T_{nr} \leq T_{load}^{const} \tag{19}$$

The heat power supplied by resources and consumed by loads are calculated and balanced in Eq. (9), with the temperature difference between mass flow in and out of node (branch). Eq. (10) and (11) calculate the temperature loss in each pipe by defining coefficients of heat loss E. Eq. (12) represents the temperature mixing process by different mass flows in each node, which means that mass flows out of a node (T_{start}, T_o) are in the same temperature after inflow with different temperature (T_{end}, T_s) are mixed together. Eq. (13)~(17) denotes the relationship between node temperature T_{ns}, T_{nr} and branch temperature T_{start}, T_{end}, T_s, T_o as well as some constants (T_{load}^{const}, T_{res}^{const}). Eq. (18) and (19) limits the node temperature to a rational range.

D. *CHP units and Objective Function*

The feasible operation region of CHP units can be described as polyhedrons [18]. The fuel cost of CHP unit *i* is expressed by a quadratic form as below:

$$f_i(p_i, h_i) = a_1 p_i^2 + a_2 p_i + b_1 h_i^2 + b_2 h_i + c_1 p_i h_i + c_2 \tag{20}$$

Thus the objective function of proposed model, i.e. minimizing total fuel cost of DHN is expressed as:

$$\min \sum_i f_i(p_i, h_i) \quad i \in \varphi^{CHP} \tag{21}$$

Eq. (1)~(19) are included as constraints in the proposed optimal dispatch model for DHN.

III. Case Studies

A. *Case Condition*

The proposed model is applied to solve a practical case in this section. As illustrated in Figure 2, this is a DHN with 20 nodes and 19 pipelines. There are 3 CHP units, 13 nodes with load and 7 nodes without load or resources. There are also 19 branches in supply network and another 19 branches in return network symmetrically. The supply temperature of each CHP units ($T_{s,i}$ $i \in \emptyset^{res}$) and return temperature of load nodes ($T_{o,i}$ $i \in \emptyset^{load}$) are assumed to be constant values (T_{res}^{const} and T_{load}^{const}, respectively). Total heat demand is 141.4 MW, and maximum heat power output of CHP units is 240 MW. Detailed data can be obtained from [21].

It should be noted that we only consider 1-hour optimal dispatch for DHN in this case study, to explore the convergence performance of proposed model by different interior-point algorithm.

There are three kinds of interior-point methods being compared:

- Traditional interior-point method (TIPM) with BFGS (quasi-newton method) as sub-problem algorithm.
- Traditional interior-point method with Conjugate gradient as sub-problem algorithm.
- Primal-dual interior-point method[22] (PDIPM)

As we all know, Newton-Raphson method will consume too much time in calculating Hessian matrix if the topology of DHN is larger. Therefore, BFGS (quasi-newton method) and conjugate gradient are applied and compared as sub-problem algorithm in the case studies. The load data are also changed with ±10% to compare the performance in different cases.

B. *Results Analysis*

The optimal dispatch results are demonstrated in Figure 2~ Figure 5 and Table 1. Figure 2 shows the topology of DHN, location and capacity of CHP and heat loads. There are also mass flow direction and exact numbers illustrated with a capital letter "M" in the front, e.g. "4 /M 270" at the center means mass flow in pipe 4 is 270 kg/s. Besides that, thickness of the line represents the size of the mass flow.

Figure 3~ Figure 5 show the power output of CHP units, mass flow in each pipe and nodes' temperature, respectively.

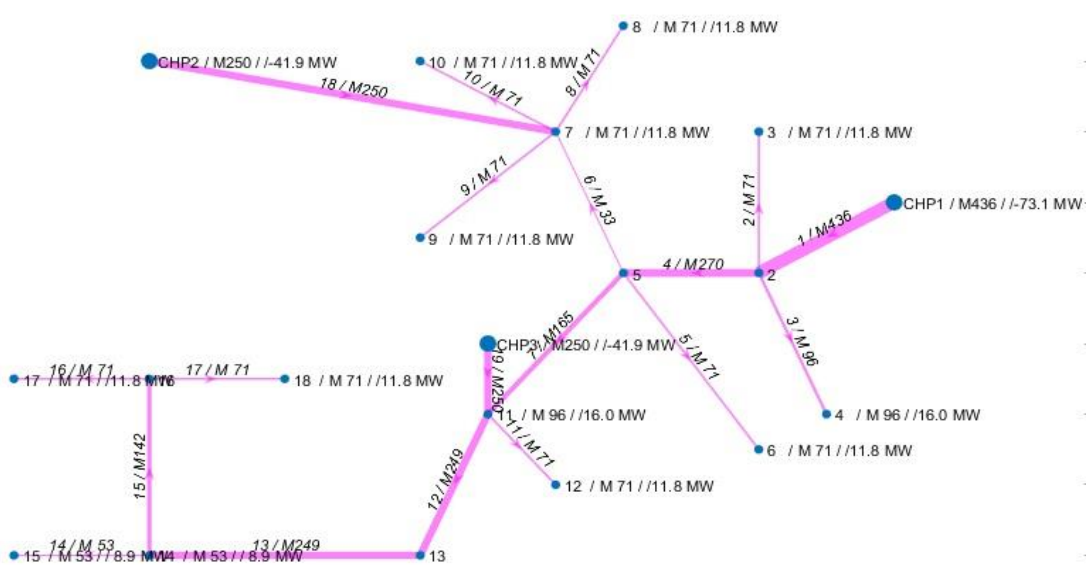

Figure 2 Optimal heat power flow result in DHN in one case (110% load with traditional interior-point method BFGS)

Table 1 Comparison between different algorithms

	Sub problem algorithm	Case 1 (110% load)		Case 2 (90% load)		Case 3 (100% load)	
		Optimal value ($)	Calculation Time (s)	Optimal value ($)	Calculation Time (s)	Optimal value ($)	Calculation Time (s)
Traditional IPM	BFGS	7420	36.82	6864	41.13	7089	4.14
	Conjugate Gradient	7400	75.49	Did Not converge	——	7080	59.92
Primal-dual IPM	BFGS	7420	5.26	7272	3.21	7820	2.69

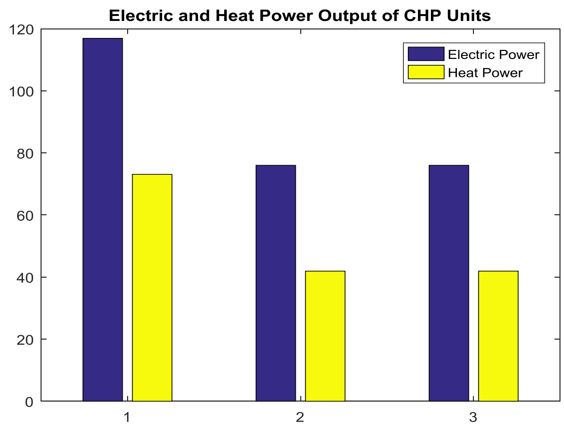

Figure 3 Optimal dispatch result of CHP units in one case (unit:MW)

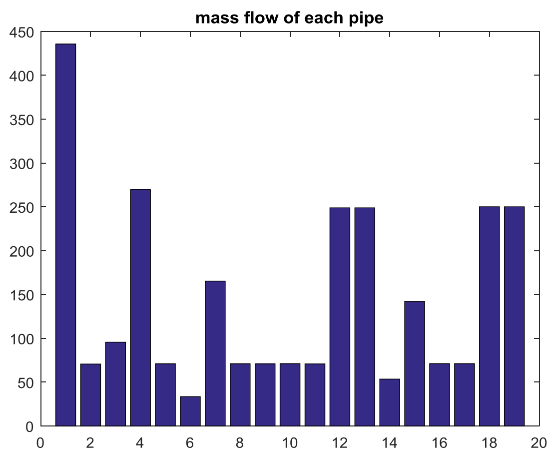

Figure 4 Mass flow rate of pipes (unit: kg/s)

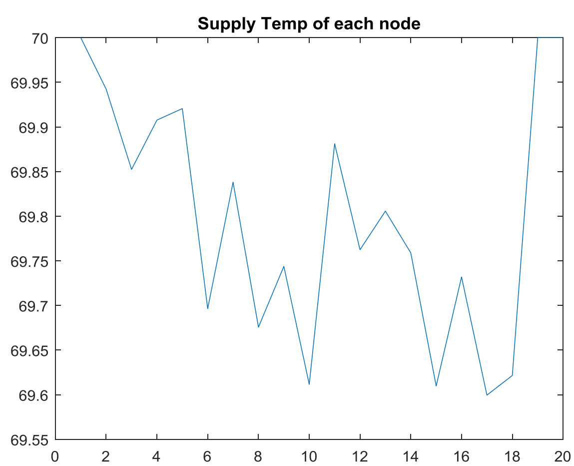

Figure 5 Supply Temperature of nodes (unit: ℃)

Table 1 showed the comparison for results of different cases and algorithm. It can be concluded from this table that:

- **Convergence speed:** PDIPM is often better than TIPM with BFGS and Conjugate Gradient (CG) as sub-problem algorithm. TIPM with CG is always the slowest and even facing the difficulty of poor convergence, e.g. case 2.
- **Quality of optimum:** in case 2 and case 3, PDIPM converge to a local optimum, which is not as good as TIPM. It's interesting that even though we spend same calculation time trying to find better optimum, the PDIPM still intend to converge to a local optimum, which is not better than TIPM. It should be noted that the result of CG (if local optimum is found) is often the best, but the difference is ignorable.

C. *Discussion*

The proposed model is a bi-convex programming problem[23] with bi-affine constraints Eq. (9)~(12), which is absolutely non-convex, thus global optimum and even convergence to a local optimum cannot be guaranteed with IPM. There are several useful suggestions for this kind of problem: Firstly, a better initial guess may help the convergence of optimization. Secondly, specific algorithm[23] designed for bi-convex optimization can be further explored to realize better convergence in proposed model. Thirdly, the model can be recast into a linear programing problem by fixing mass flow variables[14], which is also a possible solution for solving bi-convex problem iteratively.

IV. Conclusion

An optimal dispatch model for DHN with CHP units is proposed in this paper, which considers the thermal balance and hydraulic equations in the pipelines, ignoring the electric power consumed by water pump. The objective is to minimize the fuel cost in a quadratic form. This is a nonlinear bi-convex optimization problem and can be solved by interior-point method (IPM). Proposed model is validated by a case study with 20-nodes DHN and 19-branches, in which the effectiveness is proved. Furthermore, convergence of this model is explored by comparing calculation efficiency of traditional IPM and primal-dual IPM. The calculation results show that PDIPM converged faster than TIPM, but the obtained local optimum is often not better than TIPM. The feature of bi-convexity in this model needs to be further studied in the future to realize more efficient optimization for IES.

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