Stochastic Optimal Energy Management of Smart Home with PEV Energy Storage

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***Abstract*—This paper proposes a stochastic dynamic programming framework for the optimal energy management of a smart home with plug-in electric vehicle (PEV) energy storage. This work is motivated by the challenges associated with intermittent renewable energy supplies and the local energy storage opportunity presented by vehicle electrification. This paper seeks to minimize electricity ratepayer cost, while satisfying home power demand and PEV charging requirements. First, various operating modes are defined, including vehicle-to-grid (V2G), vehicle-to-home (V2H), and grid-to-vehicle (G2V). Second, we use equivalent circuit PEV battery models and probabilistic models of trip time and trip length to formulate the PEV to smart home energy management stochastic optimization problem. Finally, based on time-varying electricity price and time-varying home power demand, we examine the performance of the three operating modes for typical weekdays.**

***Index Terms*—Vehicle to Grid, Energy Management, Stochastic Dynamic Optimization, Smart Home, Plug-in Electric Vehicle.**

NOMENCLATURE

Δt	The time-step
c	The time-varying electricity price
d	The trip distance
E_{ff}	The overall electric drive efficiency
I	Current
$I^{\max}$	The maximal current
$I^{\min}$	The minimal current
$I^{\min}_{bat}$	The physical limits of battery discharging current
k	Time index
m_{hg}	The probability that pluggging-in SOC $SOC_{pi} = SOC_h$, given plugging-out SOC $SOC_{po} = SOC_g$
p	The transition probability of plugging-out
P_{batt}	The PEV battery power
P_{dem}	The power demand of the house
P_{grid}	The electric power from the grid
q	The transition probability of plugging-in
Q_{cap}	The charge capacity
Q_{eap}	The energy capacity
R_{int}	The internal resistance
S	The PEV state
SOC	The battery state-of-charge
$SOC^{\max}$	The maximal SOC
$SOC^{\min}$	The minimal SOC
$SOC^{\min}_c$	The constant minimal SOC
SOC_g	The sample values from the discretized set of feasible SOC values
SOC_h	The sample values from the discretized set of feasible SOC values
SOC_{init}	The initial SOC
SOC_{pi}	The SOC at plugging-in time
SOC_{po}	The SOC at plugging-out time
t_a	The plugging-in time
t_d	The plugging-out time
V_{oc}	The open circuit voltage

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I. INTRODUCTION

A. Motivation and Background

PEV energy storage provides a compelling opportunity to address several challenges for the security and economic sustainability of our energy supply [1]. These challenges include renewable energy integration, distributed energy resources, resilience during natural/man-made disasters (e.g. the tsunami-induced Fukushima nuclear meltdown), and rapidly evolving markets for demand-side management. If left unmanaged, however, PEVs can exacerbate peak loads and overstress local distribution circuits, resulting in less stable electricity supply and higher costs ultimately passed on to the consumer. As a consequence, researchers have recently focused on developing effective control strategies for integrating PEVs into building loads and the grid.

B. Literature Review

Researchers have examined PEV charging schedule designs for objectives such as ancillary services, frequency regulation, battery health, and effective utilization of renewable energy. The application of aggregators to frequency regulation by making fair use of PEVs' energy storage capacity is addressed in [2], [3]. Vehicle to Grid (V2G) ancillary services, such as load regulation and spinning reserves are studied in [4] by incorporating probabilistic vehicle travel models, time series pricing, and reliability. The cost of PEV battery wear due to V2G applications is presented in [5], [6]. Renewable integration is considered in [7], which derives optimal PEV charging schedules based on predicted photovoltaic output and electricity consumption.

The literature provides various V2G energy management approaches [8], [9], which can be generally categorized into linear programming (LP) [10], [11], dynamic programming (DP) [12]–[14], convex programming (CP) [15], [16], model predictive control (MPC) [17], and game theoretic approaches [18], [19]. An optimal centralized scheduling method to jointly control home appliances and PEVs is constructed as a mixed integer linear program (MILP) in [11]. A globally optimal and locally optimal scheduling scheme for EV charging and discharging are proposed using convex optimization in [16]. Energy management system for smart grids with PEVs based on hierarchical model predictive control (HiMPC) is presented in [17]. The problem of grid-to-vehicle energy exchange between a smart grid and plug-in electric vehicle groups (PEVGs) is studied using a noncooperative Stackelberg game in [18]. A strategic charging method for plugged in hybrid electric vehicles (PHEVs) in smart grids are introduced based on a game theoretic approach in [19]. Rayati *et al.* [20] present a smart energy hub for a residential customer and use reinforcement learning and Monte Carlo estimation to find a near optimal solution for the energy management problem. These approaches all share a common goal, namely, to meet overall home electric power demand while optimizing a metric such as electricity consumption, reliability, or frequency regulation.

Most of this literature pursues a V2G technology potential evaluation objective. Few pursue a real-time control system that optimizes energy management with an explicit consideration for stochastic home loads and mobility patterns. Iverson *et al.* consider probabilities of vehicle departure time and trip duration to formulate a stochastic dynamic programming (SDP) algorithm to optimally charge an EV based on an inhomogeneous Markov chain model [21]. Donadee *et al.* [22] use stochastic models of (i) plug-in and plug-out behavior, (ii) energy required for transportation, and (iii) electric energy prices. These stochastic models are incorporated into an infinite-horizon Markov decision process (MDP) to minimize the sum of electric energy charging costs, driving costs, and the cost of any driver inconvenience. A later study by [14] constructs a Markov chain to model random prices and regulation signal and formulates a SDP to optimize the charging and frequency regulation capacity bids of an EV. The previous three studies, however, do not consider integrated PEV charging with building loads. Liang *et al.* [23] provide a comprehensive literature survey on stochastic modeling and optimization tools for microgrids and examine the effectiveness of such tools.

C. Main Contribution

The main contribution of this paper is to model PEV energy storage availability by incorporating multiple random variables into a SDP control formulation of smart home energy management. The random variables include PEV arrival time, departure time, and energy required for mobility. We also quantify the potential cost savings of various operating modes, including V2G, V2H, and G2V. Our procedure provides a systematic methodology to quantify the potential cost savings to home ratepayers in smart homes with PEV energy storage.

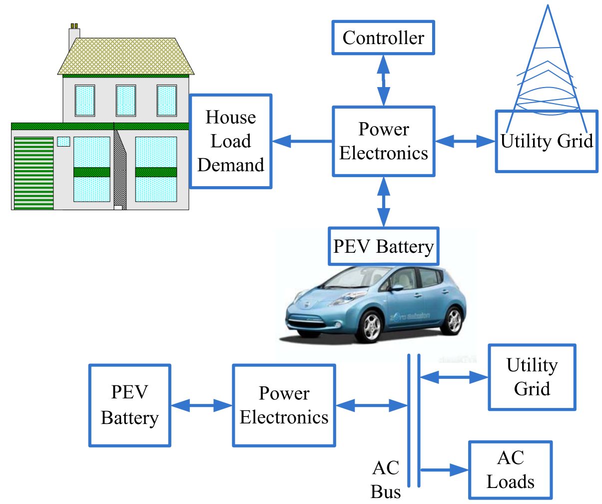


Fig. 1. Structure of PEV to smart home.

D. Outline

The remainder of the paper is arranged as follows. Section II details the models of PEV to smart home and the random variables. The SDP framework is described in Section III. The optimization results are discussed in Section IV followed by conclusions presented in Section V.

II. PEV to Smart Home Model Development

A. Operating Modes

We consider a smart home with PEV energy storage as shown in Fig. 1. The smart home is composed of the house load demand, the utility grid, a PEV with a Li-ion battery pack, and associated power electronics. The PEV battery interfaces with the utility grid and house loads via power electronics, namely a DC/AC converter. The power electronics are designed and controlled to allow bidirectional or unidirectional power flow according to the different operating modes. The controller is used to manage the power flow between the battery, house appliances and utility grid. We apply a stochastic optimal control approach to synthesize an energy management controller.

There are four operating modes in PEV to smart home systems, as shown in Fig. 2. Mode A allows PEV battery charging only (uni-directional power flow), called grid-to-vehicle (G2V). Mode B allows battery charge and discharge with the home, but does not export power to the grid. This is called vehicle-to-home (V2H). Mode C allows battery charge and discharge and may sell power to the grid, called vehicle-to-grid (V2G). The V2G mode with renewable energy (such as solar rooftop photovoltaics or wind power) is considered in mode D. Renewable energy sources will not be considered in this paper, however it is a direct extension of the proposed framework and a topic for future investigation.

To develop and evaluate our control methods, we consider load data from a single family home in Los Angeles, California. The collected data corresponds to date range 2013-04-01 to 2014-03-31. Figure 3 plots the hourly average electricity

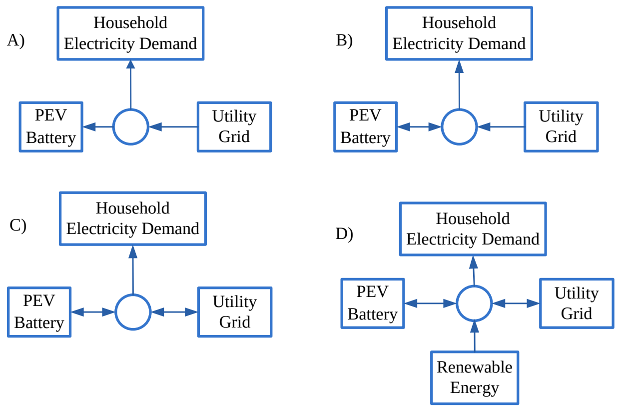


Fig. 2. Operating modes of PEV to smart home: A) grid-to-vehicle (G2V); B) vehicle-to-home (V2H); C) vehicle-to-grid (V2G); D) V2G with renewable energy generation.

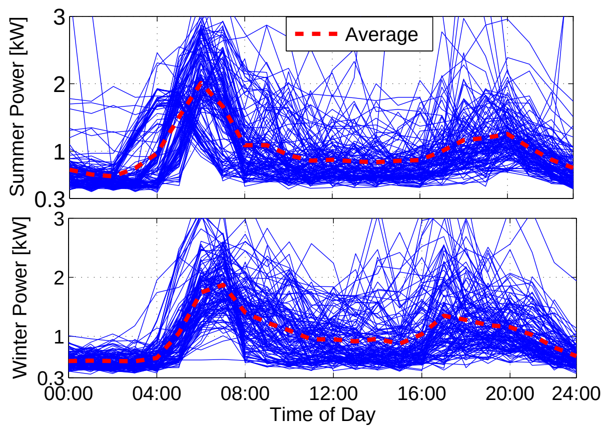


Fig. 3. Hourly electric power demand on each weekday (blue) and power demand average across year (red).

consumption on summer (May 1 - Oct 31) and winter (Nov 1 - Apr 30) weekdays.

B. Model of PEV to Smart Home

At all points in time, the system must sastify the following power balance equation,

$$P_{grid,k} = P_{dem,k} + S_k P_{batt,k}, \qquad k = 0, ..., N-1, \quad (1)$$

$$S_k = \begin{cases} 0 & \text{for } t_d \le k \le t_a \\ 1 & \text{otherwise,} \end{cases} \quad (2)$$

where S_k denotes the PEV state at time k, i.e., plugged-in ($S_k = 1$) or plugged-out ($S_k = 0$). In this paper we assume the PEV plugs-out and plugs-in once per day, each.

We consider the following discrete-time equivalent circuit model of a PEV battery

$$SOC_{k+1} = SOC_k + \frac{\Delta t}{Q_{cap}} I_k, \quad k = 0, ..., N-1 \quad (3)$$

$$SOC_0 = SOC_{init} \quad (4)$$

Consequently, we can compute the power of the PEV battery as

$$P_{batt,k} = V_{oc}(SOC_k) I_k + R_{int} I_k^2 \quad (5)$$

The charge power is assumed to be positive, by convention. In this paper, we assume the internal resistance R_{int} is constant and the open circuit voltage V_{oc} is a function of SOC_k [24].

C. Trip Time Model

PEV battery storage provides a unique opportunity to decouple energy supply from demand. A unique challenge in smart homes, however, is uncertainty in three parameters: PEV plug-in time, plug-out time, and charge required for mobility. Given statistics for these uncertain parameters, we model the PEV plug-state as a Markov chain. A Markov chain model is a dynamic system that undergoes transitions from one state to another on a state-space. Unlike deterministic dynamical systems, the process is random and each transition is characterized by statistics. Moreover, it contains the Markov property that given the present state, the future and past states are independent. Considering the PEV is plugged-in ($S_k = 1$) or plugged-out ($S_k = 0$) at time k, the Markov chain model can be written mathematically as

$$\begin{aligned} P_{ij,k} &= \Pr[S_{k+1} = j | S_k = i, k], \qquad i, j \in \{0,1\}^2, \\ P_{10,k} &= \Pr[S_{k+1} = 0 | S_k = 1, k] = p(k), \\ P_{11,k} &= \Pr[S_{k+1} = 1 | S_k = 1, k] = 1 - p(k), \\ P_{01,k} &= \Pr[S_{k+1} = 1 | S_k = 0, k] = q(k), \\ P_{00,k} &= \Pr[S_{k+1} = 0 | S_k = 0, k] = 1 - q(k). \end{aligned} \quad (6)$$

These dynamics are visualized by the state transition diagram in Fig. 4. The quantity $p(k)$ is the transition probability of plugging-out and $q(k)$ is the transition probability of plugging-in.

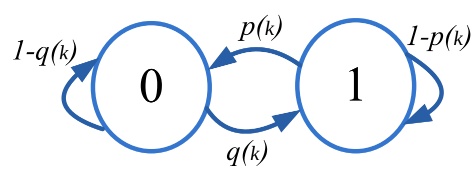


Fig. 4. State transition diagram for stochastic plug-in/out state S_k.

The start time of outgoing trips from home (or residential area) is called the plugging-out time, and the plugging-in time is the end of the last return trip. In order to research the randomness of trip time, we investigated 10 individuals daily driving schedules over 3197 person-work days. All 10 people work in a university office in Chengdu, China and their work hours are from 8:30 AM to 5:30 PM. Chengdu is a capital city in the southwest of China and the core city resident population is about 5.65 million. According to the analysis of the daily driving schedules, the temporal distribution of vehicle transition probability is shown in Fig. 5. The plugging-out time distribution is concentrated around 6:45-8:30 AM,

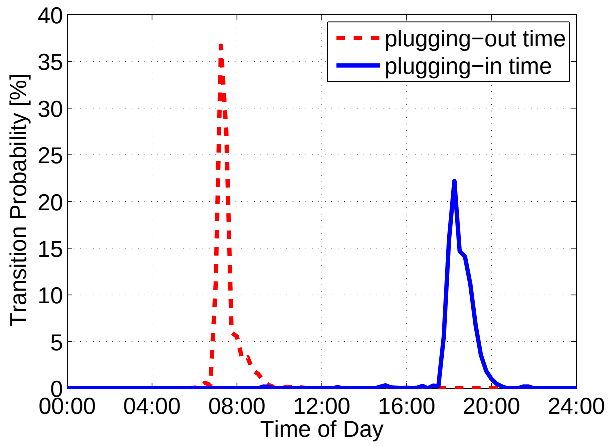


Fig. 5. Distribution of plugging-in and plugging-out times for 10 individuals over 3197 person-work days in Chengdu, China.

and corresponds to morning commutes. The mean value of the plugging-out time is 7:40 AM (7.66h), and the standard deviation (std) is 0.57h. The plugging-in time distribution shows the highest peak around 5:30-8:00 PM, the mean value is 6:38 PM (18.64h), and the std is 0.89h.

D. SOC at Plugging-in Time Model

In this section, we use daily trip distance to compute the SOC at plugging-in time, denoted SOC_{pi}. The randomness of SOC at plugging-in time is affected by many factors, including the SOC at plugging-out time, driving distance, driving styles, route choice, traffic, etc. Here we only consider the effect of driving distance

$$SOC_{pi} = \begin{cases} SOC_c^{\min}, & \text{if } SOC_{po} - \frac{d}{E_{ff} \cdot Q_{eap}} \le SOC_c^{\min}, \\ SOC_{po} - \frac{d}{E_{ff} \cdot Q_{eap}}, & \text{otherwise}, \end{cases} \tag{7}$$

where Q_{eap} is the energy capacity [kWh] and E_{ff} is the overall electric drive efficiency which we assume equal to 6.7km/kWh [25]. Random variables SOC_{po} and d represent the SOC at plugging-out time (see Fig. 5) and daily driving distance. If given SOC_{po} and d, then SOC_{pi} can be computed. Note that SOC_{pi} is lower-bounded by $SOC_c^{\min}$, which prevents battery depletion. Consequently, we can compute the conditional probability distribution of SOC_{pi} according to

$$m_{hg} = Pr[SOC_{pi} = SOC_h | SOC_{po} = SOC_g], \tag{8}$$

$$SOC_h, SOC_g \in \mathcal{S} = \{SOC_i = SOC_c^{\min} + i \cdot \Delta SOC \mid i \in \mathbb{N},\ SOC_c^{\min} \le SOC_i \le SOC^{\max}\}. \tag{9}$$

The quantity m_{hg} is the probability that plugging-in SOC $SOC_{pi} = SOC_h$, given plugging-out SOC $SOC_{po} = SOC_g$. When given a plug-out SOC_{po} ($SOC_{po} = SOC_g$), the probability distribution of SOC_{pi} is decided by the probability distribution of driving distance. According to the statistical daily trip length distribution from 2009 U.S. National Household Travel Survey (NHTS) [26], the conditional probabilities for the plugging-in SOC SOC_{pi}, given the plugging-out SOC

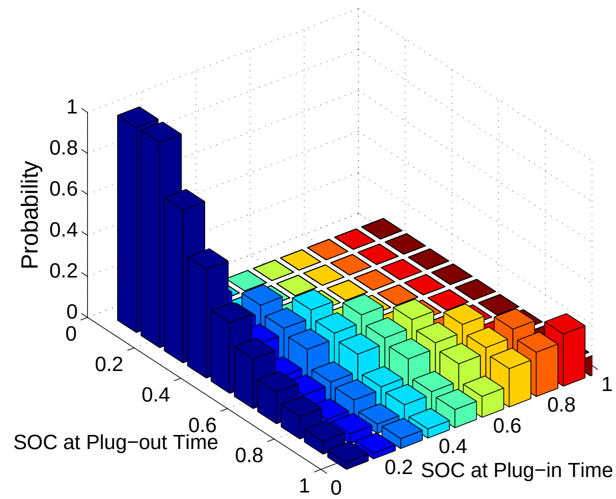


Fig. 6. Conditional probabilities of plugging-in SOC SOC_{pi}, given the plugging-out SOC SOC_{po}.

SOC_{po} are shown in Fig. 6. Note the SOC at plugging-in time is always less than or equal to SOC at plugging-out time - that is SOC cannot increase during driving events.

III. Stochastic Dynamic Programming

This section presents the stochastic dynamic programming approach used for solving the optimal power management problem for PEV to smart home microgrid. The objective is to manage power flow to minimize electricity cost. The electricity cost includes household electric power demand, PEV battery charging and discharging. This objective is oriented toward individual homeowners. Other objectives are directly applicable as well, e.g. minimize marginal power plant carbon emissions, battery degradation, or distribution circuit voltage drop. To explicitly compute the electricity cost for a PEV to smart home microgrid, we define the instantaneous electricity cost functional, $g_k(SOC_k, S_k, I_k)$, as follows:

$$\begin{aligned} g_k(SOC_k, S_k, I_k) &= c_k \cdot \Delta t \cdot P_{grid,k} \\ &= c_k \cdot \Delta t \cdot (P_{batt,k} S_k + P_{dem,k}) \\ &= c_k \cdot \Delta t \cdot ((V_{oc}(SOC_k) I_k + R_{int} I_k^2) S_k + P_{dem,k}) \end{aligned} \tag{10}$$

where c_k is the time-varying electricity price [cents/kWh]. In this paper we assume deterministic home power demand. While this assumption is clearly never true, the literature is rich with machine learning and stochastic modeling approaches for home power demand [27], [28], that can be incorporated into our framework. As such, we focus on the novel aspects of this work - modeling PEV energy storage availability.

The controller must maintain battery SOC_k and current I_k within simple bounds,

$$SOC^{\min} \le SOC_k \le SOC^{\max}, \quad k = 0, ..., N \tag{11}$$

$$I^{\min} \le I_k \le I^{\max}, \quad k = 0, ..., N-1 \tag{12}$$

The various operating modes (see Fig. 2) are incorporated by appropriately setting values of $I^{\min}$.

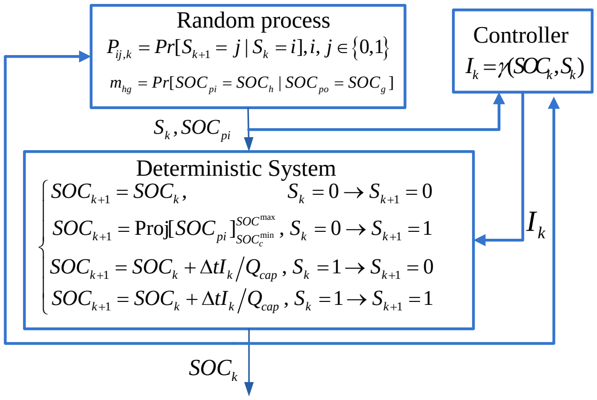


Fig. 7. Block diagram of stochastic multi-stage decision process.

i) In V2G mode (Mode A),

$$I^{\min} = I_{bat}^{\min}, \tag{13}$$

ii) In V2H mode (Mode B),

$$I^{\min} = \max\left\{ I_{bat}^{\min}, \frac{-V_{oc} + \sqrt{V_{oc}^2 - 4R_{int}P_{dem,k}}}{2R_{int}} \right\}. \tag{14}$$

iii) In G2V mode (Mode C),

$$I^{\min} = 0. \tag{15}$$

Armed with the Markov chain modelling framework to incorporate statistics of the random process (e.g. plugging-out time, plugging-in time, SOC at plugging-in time), we can now formulate a stochastic dynamic program (SDP). Consider the block diagram in Fig. 7. The deterministic subsystem is given in the lower-left block, and is characterized by state SOC_k. The stochastic subsystem is characterized by the pair of states $\{S_k, SOC_{pi}\}$, described by the Markov chain model in the top block. The design problem is to determine the control input I_k which minimizes the electricity cost. The control will be synthesized as a time-varying state feedback control law. Namely, the control is the output of a mapping that depends on the current deterministic state SOC_k and stochastic state $\{S_k, SOC_{pi}\}$. We formalize this as a finite-time stochastic dynamic program [29],

$$\min_{I_k, SOC_k, S_k} \mathbb{E} \sum_{k=0}^{N-1} c_k \cdot \Delta t \cdot (S_k P_{batt,k} + P_{dem,k}) \tag{16}$$

s. t.

$$SOC_{k+1} = \begin{cases} SOC_k, S_k = 0 \rightarrow S_{k+1} = 0 \\ \text{Proj}[SOC_{pi}]_{SOC_c^{\min}}^{SOC^{\max}}, S_k = 0 \rightarrow S_{k+1} = 1 \\ SOC_k + \frac{\Delta t}{Q_{cap}} I_k, S_k = 1 \rightarrow S_{k+1} = 0 \\ SOC_k + \frac{\Delta t}{Q_{cap}} I_k, S_k = 1 \rightarrow S_{k+1} = 1 \end{cases}$$

$$\text{Eqns } (1)-(9) \quad \text{and} \quad (11)-(15). \tag{17}$$

Now we define the value function. Let $V_k(SOC_k, S_k)$ be the minimum expected cost-to-go from time step k to N, given the current battery SOC level and the plug-state - SOC_k, S_k, respectively. Then the principle of optimality [29] is given by:

$$\begin{aligned} V_k(SOC_k, S_k) &= \\ &\min_{I_k \in \mathcal{D}_k} \{g_k(SOC_k, S_k, I_k) + \mathbb{E}\ V_{k+1}(SOC_{k+1}, S_{k+1})\} \\ &= \min_{I_k \in \mathcal{D}_k} \{g(SOC_k, S_k, I_k) + \\ &\sum_{j \in \{0,1\}} P_{ij,k}\ V_{k+1}(SOC_{k+1}, S_{k+1} = j)\}, \end{aligned} \tag{18}$$

where $g_k(\cdot,\cdot)$ is the instantaneous cost in (10) and $P_{ij,k}$ are Markov chain transition probabilities in (6). The minimization operator is subject to a time-varying admissible control set $\mathcal{D}_k$ characterized by (1)-(9) and (11)-(15). We can further expand (18) by considering separate cases for $S_k = 0$ and $S_k = 1$ and substituting the SOC dynamics as follows.

$$\begin{aligned} V_k(SOC_k, S_k = 0) &= \\ &\min_{I_k \in \mathcal{D}_k} \{g(SOC_k, S_k = 0, I_k) \\ &+ (1 - q(k)) \cdot V_{k+1}(SOC_k, S_{k+1} = 0) \\ &+ q(k) \cdot V_{k+1}(\text{Proj}[SOC_{pi}]_{SOC_c^{\min}}^{SOC^{\max}}, S_{k+1} = 1)\Big\}, \end{aligned} \tag{19}$$

$$\begin{aligned} V_k(SOC_k, S_k = 1) &= \\ &\min_{I_k \in \mathcal{D}_k} \{g(SOC_k, S_k = 1, I_k) \\ &+ p(k) \cdot V_{k+1}\left(SOC_k + \frac{\Delta t}{Q_{cap}} I_k, S_{k+1} = 0\right) \\ &+ (1 - p(k)) \cdot V_{k+1}\left(SOC_k + \frac{\Delta t}{Q_{cap}} I_k, S_{k+1} = 1\right)\Bigg\}. \end{aligned} \tag{20}$$

We also have the boundary condition

$$V_N(SOC_N, S_N) = \begin{cases} 0, \text{for } SOC_N^{\min} \leq SOC_N \leq SOC_N^{\max} \\ \infty, \text{otherwise.} \end{cases} \tag{21}$$

Finally, the optimal control action is saved as

$$\begin{aligned} I_k^* &= \gamma_k(SOC_k, S_k) \\ &= \arg\min_{I_k \in \mathcal{D}_k} \{g(SOC_k, S_k, I_k) \\ &+ \sum_{j \in \{0,1\}} P_{ij,k} V_{k+1}(SOC_{k+1}, S_{k+1} = j\}. \end{aligned} \tag{22}$$

IV. Results & Discussion

This section analyses the SDP controller properties by comparing its performance in different operating modes. The time-varying price signal, driving cycles and average electricity power demand are input to the SDP control algorithm. First, we do the SDP multi-stage decision process offline and find the optimum charge and discharge current and minimum expected electric energy cost, for any time k and any value of SOC_k. Then, we use the optimal control policy computed by SDP to simulate the closed-loop system. The PEV is allowed to charge only at home. Table I lists the parameter values used for these optimization case studies presented in this manuscript.

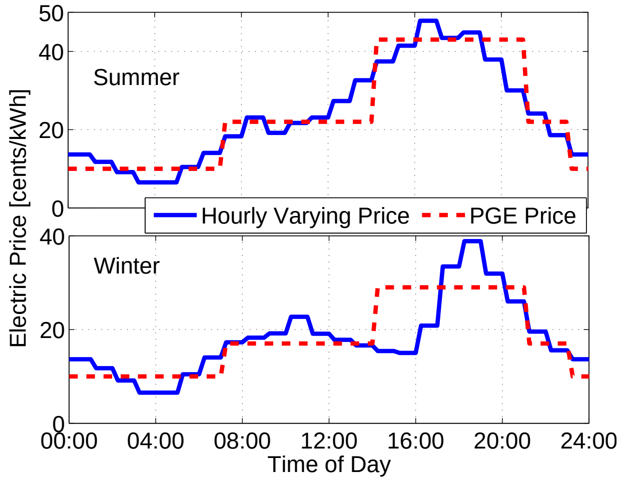


Fig. 8. Electric Price in a Day.

A. Time-Varying Electricity Price

Pacific Gas and Electric Company (PG&E) offers special EV rate plans for residential customers in different seasons. They are non-tiered, time-of-use plans shown in Fig. 8 [30]. EV charging cost is based on the time of day you consume electricity. Costs are lowest from 11 PM to 7 AM when demand is lowest. Electricity is more expensive during Peak (2-9 PM) and Partial-Peak (7 AM-2 PM and 9-11 PM) periods. Similar to the PG&E EV rate plans, we synthetically generated a more interesting hourly time-varying electric price, as shown in Fig. 8. The hourly time-varying electric prices vary between 6.5-47.8 cents/kWh in summer and 6.5-38.8 cents/kWh in winter, providing an opportunity for the PEV to gain economic benefits through off-peak charging and bidirectional PEV to smart home power exchange. This might emulate a future time-of-use EV pricing scenario that several utilities are considering. In summer, both the PG&E EV price and hourly time-varying electric price's average is 23.8 cents/kWh. In winter, average is 18 cents/kWh. In this paper, we examine the performance of PEV to smart home with the hourly time-varying electricity price.

TABLE I
SYSTEM PARAMETERS.

Parameter Description	Symbol	Value	Unit
Battery Charge Capacity	Q_{cap}	66	Ah
Battery Energy Capacity	Q_{eap}	24	kWh
Battery Open Circuit Voltage	V_{oc}	$f(SOC)$	V
Battery Internal Resistance	R_{in}	0.1	Ohm
Time Step	Δt	15	min
Maximum Battery SOC	$SOC^{\max}$	0.95	-
Constant Minimum Battery SOC	$SOC_c^{\min}$	0.075	-
Minimum Battery SOC	$SOC^{\min}$	0.075	-
Maximum Charging Current	$I_{bat}^{\max}$	20.27	A
Minimum Discharging Current	$I_{bat}^{\min}$	-20.27	A
Plugging-out Time	t_d	7:40 AM	
Plugging-in Time	t_a	6:38 PM	

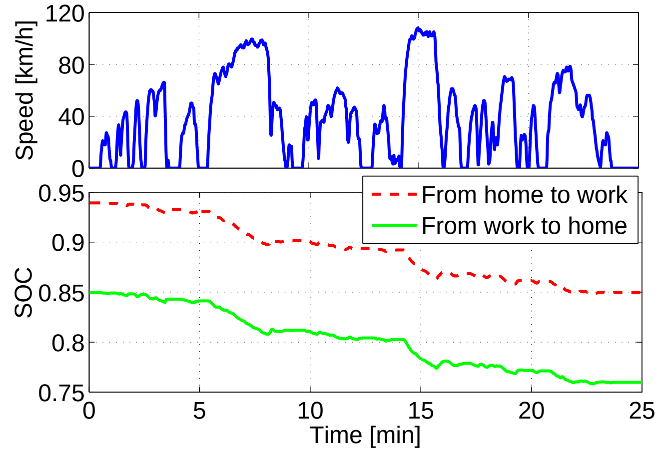


Fig. 9. SOC curves for a PEV undergoing LA92 drive cycle.

B. Average Weekday Household Power Demand

This section presents the resulting SDP control law simulated on the average weekday household power demand, average plug-out and average plug-in times. The time horizon is 24 hours. We assume the PEV is driven between work and home with one LA92 drive cycle (15.8 km and 23.93 minutes). A simple PEV energy management control strategy is considered. All the drive power is supplied by the battery and all deceleration power is captured with regenrative braking, unless: (i) the deceleration power exceeds the motor's limits; (ii) the deceleration power causes battery voltage to exceed $V_{\max}$. Then it uses friction brakes for remaining deceleration power. The SOC curves for a PEV similar to a Nissan Leaf undergoing LA92 drive cycle are shown in Fig. 9. The SOC at plugging-out time SOC_{po} is 0.94. The SOC at plugging-in time SOC_{pi} is 0.76.

Three different operating modes are investigated, including (i) V2G mode; (ii) V2H mode; (iii) G2V mode. In the first two, the battery can supply power to the home and grid. Based on average summer weekday power demand, as well as hourly time-varying electric price, the battery SOC trajectories, electric power from grid, and total electric cost are depicted in Fig. 10. Here we assume the SOCs, SOC_0, at the start time of a day (00:00) are 0.10, 0.52 and 0.76 in V2G, V2H and G2V mode, respectively, which equal to the SOC values at the end time of a day (24:00).

It is evident that the depth of battery discharge is larger under V2G mode than V2H mode. Of course, more frequent charging and discharging will affect battery life, which will be considered in the future work. Based on the summer hourly time-varying electric price, a majority of the charging occurs during the low electricity price period: 2:00-5:30 AM in V2G mode; 3:00-5:00 AM in V2H and G2V modes. A majority of the discharging during the high electricity price period: 6:30-9:00 PM.

The daily electricity cost based on average weekday power demand and LA92 drive cycle are shown in Table II. In V2G mode, the daily electricity cost is 4.50 USD less in summer

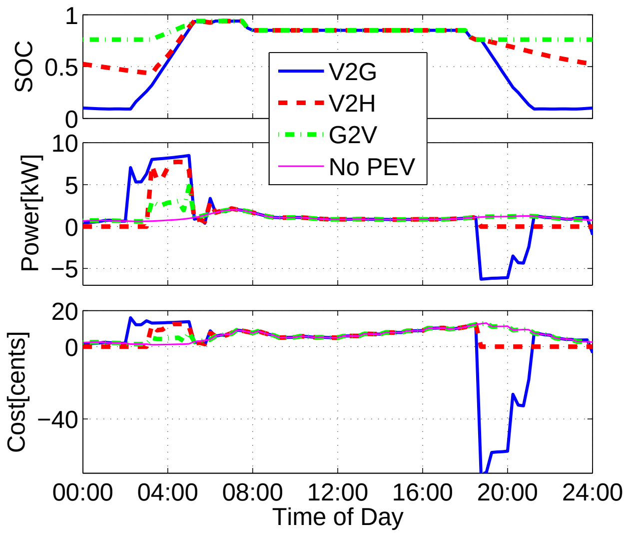


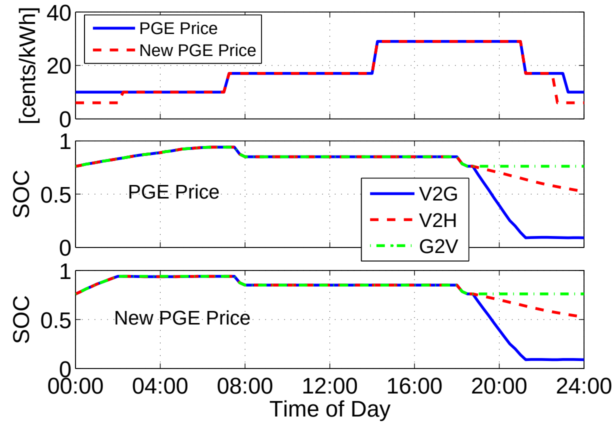
Fig. 10. SOC, grid power, and instantaneous cost trajectories of SDP controllers based on summer average weekday house power demand.

and 3.65 USD less in winter relative to the No PEV case. In V2H mode, the daily electricity cost is 1.10 USD less in summer and 0.84 USD less in winter than the No PEV case. In G2V mode, the daily electricity cost is 0.29 USD more than the No PEV case in both summer and winter. Summer is more expensive than winter due to higher electricity prices and larger household power demand. These estimates can be combined with charging infrastructure costs to calculate the best-case return-on-investment (ROI) period for each working mode, which Interested readers should pay more attention to.

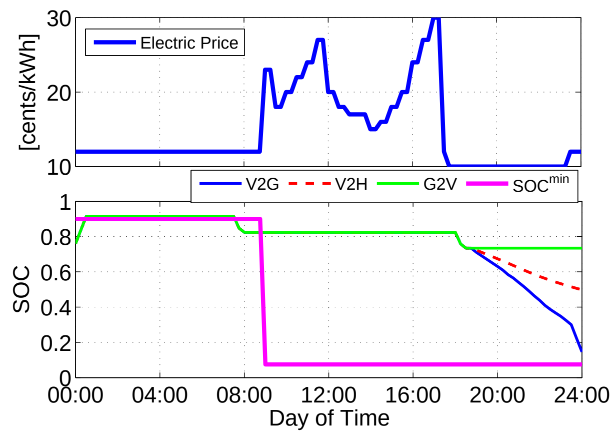
TABLE II
DAILY ELECTRICITY COST BASED ON AVERAGE WEEKDAY POWER DEMAND AND LA92 DRIVE CYCLE.

	Summer cost (USD)		Winter cost (USD)	
Modes	total	PEV	Total	PEV
V2G	1.40	-4.50	1.15	-3.65
V2H	4.81	-1.10	3.94	-0.84
G2V	6.19	+0.29	5.07	+0.29
No PEV	5.90	0	4.78	0

Based on the previous results, it would seem necessary for the time-of-use price to be less before morning departure than the price after evening arrival to ensure sufficient SOC for mobility needs. This time-of-use price characteristic is not necessary, as demonstrated in Fig. 11(a). Namely, we have considered a PG&E price scheme, along with a new (i.e. modified) PG&E price scheme. The new scheme has lower prices after evening arrival than prices before morning departure. Yet, the SOC increases. We have observed, however, that for some other electricity price structures the proposed SDP will not sufficiently charge the PEV to meet mobility constraints. The reason for this behavior might be due to numerical or implementation issues in the simulation. The decisions of charging



(a)



(b)

Fig. 11. Battery SOC if the evening electric prices are less than or equal to the morning electric prices.

and discharging is decided by time varying electric price, transition probability of plugging-out, constraints of current and SOC. Here we consider one practical heuristic solution by making $SOC^{\min}$ time-varying. That is, make $SOC_k^{\min} = 90\%$ when there's some non-trivial probability of plugging-out, as $SOC_k^{\min} = 90\%$ for k's where $P_{10,k} = Pr[S_{k+1} = 0 | S_k = 1, k] > 1\%$. $SOC_{k-u}^{\min} = 90\%$, $u = 0, 1, ..., 30$ for the first $P_{10,k} = Pr[S_{k+1} = 0 | S_k = 1, k] > 1\%$. To illustrate, we consider average summer house power demand, LA92 drive cycle, the new electricity price, and time-varying $SOC^{\min}$ as shown in Fig. 11(b). The battery SOC in V2G, V2H and G2V mode are shown in Fig. 11(b). This time-varying minimum SOC ensures the SOC is sufficiently high to meet mobility demands - in all modes.

C. Impact of Varying Mobility Needs

This section investigates the impact of varying daily commute distances on smart home power management. We examine daily trip lengths between 5 km and 150 km, which includes the bulk of daily trip length distribution according to the NHTS data. In Table III, several drive cycles, trip lengths and SOC for driving are reported. The total electricity cost, PEV battery electric cost and SOC value at the end of the day

TABLE III
DRIVE CYCLES CONSIDERED TO ASSESS IMPACT OF DAILY TRIP LENGTH ON SMART HOME ENERGY MANAGEMENT.

Cycles	Daily Trip Length/km	SOC_d
2x(HWFET+LA92+US06+UDDS+NEDC+SC03)	147.58	0.84
2x(HWFET+LA92+US06+UDDS+SC03)	125.72	0.76
2x(LA92+US06+UDDS+NEDC)	103.22	0.61
2x(LA92+US06+UDDS)	81.16	0.49
2x(HWFET+US06+SC03)	70.32	0.47
2x(US06+UDDS+SC03)	61.28	0.37
2x(LA92+NEDC)	53.16	0.30
2x(LA92+SC03)	43.12	0.23
2xHWFET	33.02	0.21
2xLA92	31.60	0.18
2xUS06	25.78	0.21
2xUDDS	23.98	0.11
2xNEDC	21.86	0.12
2xSC03	11.52	0.05
SC03	5.76	0.03

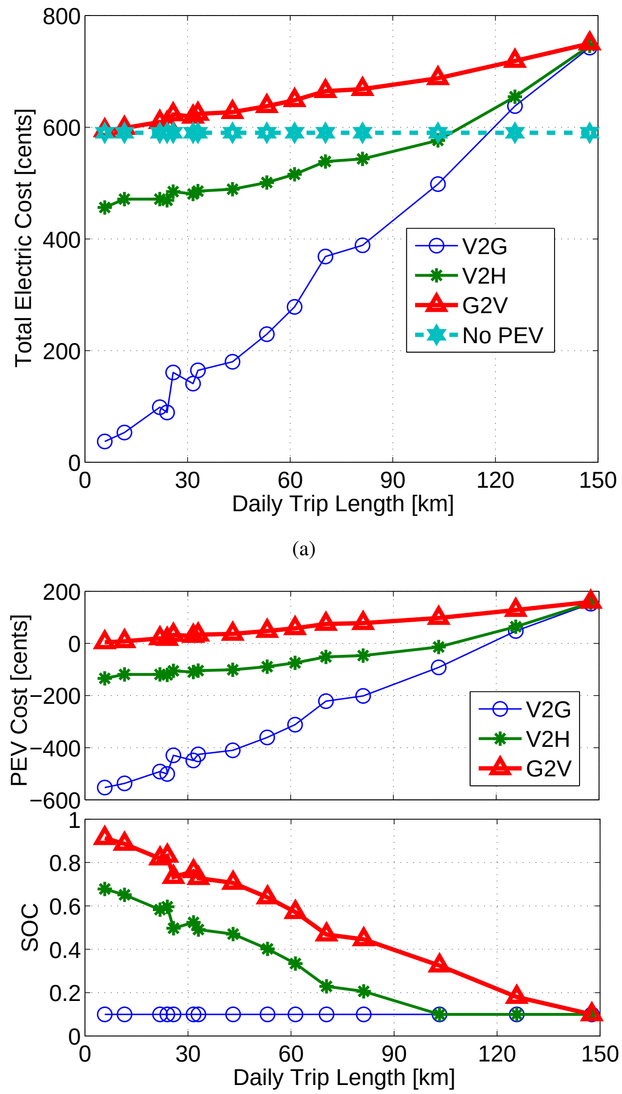
(24:00) in summer weekday with different daily trip are shown in Fig. 12. The total electric costs and PEV electric costs increase as daily trip length increases, for all three operating modes. When the daily trip length equals the PEV electric range, then the costs are the same in all three modes. In G2V mode, the SOC values (at 24:00) decrease as daily trip length increases. In V2H mode, the SOC values (at 24:00) decrease as daily trip length increases, for trip lengths less than 103 km, but are constant as the trip length increasing. In V2G mode, the SOC values (at 24:00) are constant for all daily trip lengths. From Fig. 12-(b), it can be seen that PEVs in V2H or V2G mode provide net cost reductions when daily trip length is less than 100 km.

D. Financial Analysis

To demonstrate the potential financial benefits of PEV to smart home microgrids, this section considers the annual profits for different operating modes. In this part, we use the 2xLA92 drive cycles as the daily trip. The daily electric cost under no PEV case, V2G, V2H, and G2V modes are presented in Fig. 13. The analysis summarized by Table IV examines the annual electric cost in summer and winter.

In summer weekdays, the mean value of No PEV daily electric cost is 5.91 USD/day, and the total summer weekday cost is 768.8 USD. The mean value of V2G daily electric cost is 1.42 USD, and the total summer weekday cost is 184.3 USD. The mean value of V2H daily electric cost is 4.80 USD/day, and the total summer weekday cost is 624.1 USD. The mean value of G2V daily electric cost is 6.20 USD/day, and the total summer weekdays cost is 805.8 USD. The summer weekday total electric cost is 76.0% less for V2G mode relative to No PEV case, and it is by 18.8% in V2H mode, but 4.8% more for G2V mode relative to No PEV case.

In winter weekdays, the mean value of No PEV daily electric cost is 4.61 USD/day, and the total winter weekdays cost is 599.0 USD. The mean value of V2G daily electric cost is 1.16 USD, and the total winter weekdays cost is 151.4 USD. The mean value of V2H daily electric cost is 3.87 USD, and



(a)

(b)

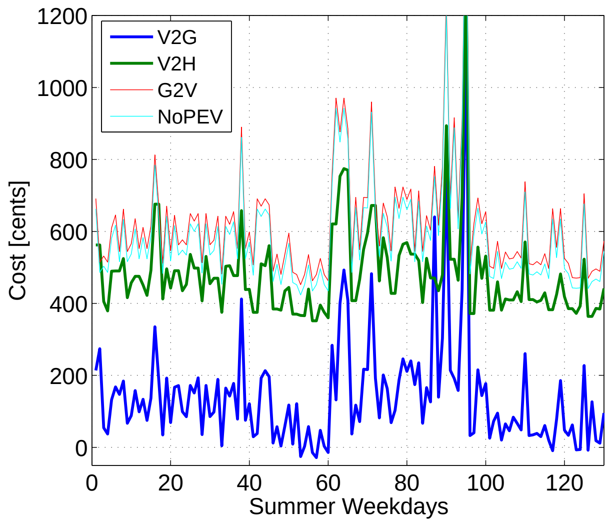
Fig. 12. Electricity cost in summer weekday with varying daily trip length.

the total winter weekdays cost is 503.0 USD. The mean value of G2V daily electric cost is 4.89 USD, and the total winter weekdays cost is 636 USD. The winter weekdays total electric cost is 74.7% less for V2G mode relative to No PEV case, and it is by 16.0% in V2H mode, but 6.2% more for G2V mode relative to No PEV case.

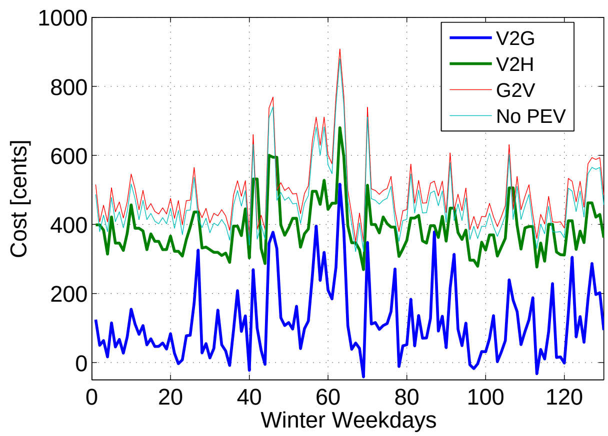
Over one year, total electric cost is 75.5% less for V2G mode relative to the No PEV case, and it is 17.6% less in V2H mode. The cost increases by 5.4% in G2V mode. The energy management strategy is exploiting price arbitrage selling electricity to the grid when its high and buying electricity from the grid when its low.

E. Discussion on Power Quality Problem

Power quality in distribution circuits is a key challenge associated with PEV charging. Without coordinated charging, PEV charging can lead to voltage loss and hence reliability concerns and increased energy costs. Coordinated charging



(a) SUMMER



(b) WINTER

Fig. 13. Electricity cost in different modes.

TABLE IV
ANNUAL ELECTRIC COST (USD) DUE TO WEEKDAY HOUSE POWER DEMAND, PEV CHARGING, AND LA92 DRIVING CYCLE.

	V2G	V2H	G2V	No PEV
Summer Daily Average	1.42	4.80	6.20	5.91
Winter Daily Average	1.16	3.87	4.89	4.61
Year Daily Average	1.29	4.34	5.55	5.26
Summer Total	184.3	624.1	805.8	768.8
Winter Total	151.4	503.0	636.0	599.0
Year Total	335.7	1127.1	1441.8	1367.8

can minimize the power losses and maximize distribution grid load factors [14], [31]. According to Fig. 10, the total power demand from the grid is reduced when grid load is high, and is increased during the night based on SDP control. The load shifting effect is significant and important to improve grid power quality. However, the maximal power demand from the grid may violate voltage deviation limits, and excessive discharge may affect grid stability. Although this manuscript is concerned with a single node, and not the distribution network, we conduct a simple power quality analysis by

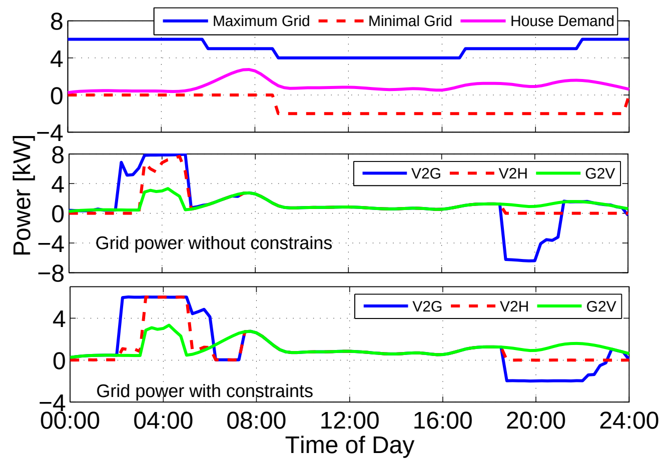


Fig. 14. SDP optimized grid power trajectories, with and without the grid power limits given in the top subplot.

considering power limits from the grid (top subplot of Fig. 14). Specifically, we consider a scenario where the utility provides day-ahead power limit signals, $P_{grid,k}^{\min} \leq P_{grid,k} \leq P_{grid,k}^{\max}$, to the controller which ensures distribution circuit power quality. Thus, we regulate the PEV charging power to avoid excessive deportation or importation of power from the local distribution grid. To illustrate, we consider house power demand on an arbitrary day during winter, the winter hourly electricity price, and the LA92 drive cycle. The results are shown in Fig. 14. With grid power limits, the magnitude of power transferred between the grid and home is less. Consequently, the charging/discharging times are elongated in V2G mode. In V2H mode, the maximum power from grid is reduced. The difference is negligible in G2V mode, since the grid power demand constraints do not become active.

V. CONCLUSION

This paper examines a stochastic optimization framework for energy management of a smart home with PEV energy storage, with a specific focus on PEV energy storage uncertainty. A stochastic dynamic programming problem (SDP) is formulated to optimize the electric power allocation among the PEV battery, home power demand, and utility grid. The strategy explicitly incorporates probability distributions of trip time and trip length. We quantify the potential cost savings of various operating modes, including V2G, V2H and G2V. We find that variable mobility patterns significantly impact the optimal energy management behavior. Additionally, significant operational cost savings are achievable with the V2G operating mode, given the electricity cost structure assumed in this paper.

Future work may incorporate thermal and aging dynamics of the PEV battery into the SDP optimization framework, since the V2G or V2H operating modes will impact battery health. The proposed framework can be extended to consider additional uncertainties, such as house power demand, time-varying electricity price, renewable power generation. Combined with charging infrastructure costs, the best-case return-on-investment (ROI) period for different working modes can be calculated.

ACKNOWLEDGMENT

This work is supported partially by Science and Technology Department of Sichuan Province (Grant Nos. 2016HH0010, 2015TD0021, and 2013JY0088), NSF of China (Grant No. 51375402), and Xihua University Fund (Grant Nos. z1220315, and szjj2012-012).

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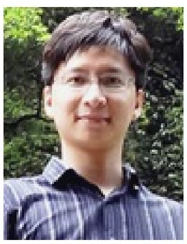

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